IDENTIFICATION OF SHIP MODEL MOTION PARAMETERS

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(1966)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF NAVAL ENGINEER

AND THE DEGREE OF MASTER

OF SCIENCE IN

MECHANICAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF

TECHNOLOGY

June 1971

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Sudmitted to the Department of Naval Architecture and Marine Engineering and the Department of Mechanical Engineering in partial fulfillment of the requirements for the degrees of Naval Engineer and Master of Science in Mechanical Engineering

ABSTRACT

The applicability of the methods of system identification to the determination of ship model motion parameters is studied taking as an example a model of the Mariner class hull form.

A mathematical expression describing the dynamical behaviour of an ocean vehicle is first presented. Then the values of the ship model motion parameters to be identified are predicted using available techniques. Data representing the dynamical behaviour of the hull form under controlled conditions is obtained with model tests in the towing tank. This data is then used together with the first estimated values of the motion parameters to determine the corrected values of those parameters.

The sensitivity of convergence to several noise factors caracteristic of the process is also studied.

The results indicate a fairly strong dependence on those noise factors and on the initial uncertainty that in general decide the final convergence. This fact seems to indicate that the particular technique used and the particular input adopted are not in conjunction the most appropriate for this type of dynamical behaviour.

Thesis Supervisor : Martin A. Abkowitz
Title : Professor of Naval Architecture

ACKNOWLEDGMENTS

The author wishes to extend his sincere thanks to his thesis supervisor, Professor Martin A. Abkowitz, for his assistance throughout the project. Appreciation is also expressed to Professor David N. Wormley for his acceptance in acting as the thesis reader for the Department of Mechanical Engineering.

The help of Mr. Ysabel Mejia of the M.I.T. Shipmodel Towing Tank in conducting the experiments and of Mr. Emile Boulos of the Charles Draper Laboratories in preparing equipment and circuits for the measurements is also sincerely appreciated.

The assistance of Mr. Thomas Deville, Mr. Michael Hayes and Mr. Fohn Odea deserve also a special thanks.

Mrs. Barbara Burke, who typed the manuscript, is also entittled to the author's appreciation.

The model tests were run at the M.I.T. Shipmodel Towing Tank.

The analysis of the data was performed at the Mechanical Engineering Computation Center and at the M.I.T. Information Processing Center.

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CHAPTER 1

INTRODUCTION

Consider a system whose inputs are known and the outputs observable. System identification, in general, has the objective of determining the structure of that system using the known inputs and measuring the outputs in order that one may be able to predict the behaviour of the system (outputs) to the various excitations expected from the environment (inputs). One such a system of interest in naval architecture is the ocean vehicle. The naval architect wants to be able to predict the velocities, accelerations and other important motion responses that a ship will experiment at sea in order to effectively design for proper response, structural integrity, and other critical aspects of operability.

The identification of a system of completely unknown structure is, in most cases, impossible, at least for the time being. There are, however, some types of systems for which identification is possible. Those consist of parameter identification problems where the general structure of system is known but the particular values for a set of parameters are to be determined. The ocean vehicle has, fortunately, these characteristics. Using rigid body dynamics, one is able to develop the so-called equations of motion which describe the dynamical behaviour of the

vehicle. In these equations there are some parameters (ship motion parameters or hydrodynamic coefficients) which are not known or at best whose value is known with a certain degree of uncertainty. It will be necessary then, in order to fully predict the behaviour of ocean vehicles at sea, to obtain the correct values of those parameters or, at least, by selected measurements, to decrease the degree of uncertainty. Unfortunately these measurements are, more often than not, combined with noise from the instruments themselves or from the environment at the time the measurements take place or from both. These types of noise need to be taken into account when identifying the parameters. An example of the foregoing is the case of experiments in a towing tank to obtain data from models to apply to full scale vehicles. Measurement noise exists whenever one uses any type of transducers that always show some hysteresis and are characterized by a specific dynamical behaviour. Environment noise will be caused among others by interference between the model and undesired waves, and vibration of test equipment. Of course these effects could be included in the mathematical model of the vehicle but it would make the analysis of its behaviour much more difficult. Thus it is seen that sometimes it's preferable

to assume a simpler model for the behaviour of the system and introduce the effects not accounted for in the mathematical model as noise (environmental noise) whose general nature is assumed known.

Many of the available techniques of system identification were developed for a dynamical system whose behaviour may be expressed in a so-called state space representation[1]:

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u},t) \;; \quad \underline{x}(t_o) = \underline{x}_o \tag{1.1}$$

$$\underline{y} = \underline{h}(\underline{x},\underline{u},t) \tag{1.2}$$

where the vector $\underline{x}$ expresses the state of the system at time t ($t > t_o$), the vector $\underline{y}$ represents the output of the system at time t and the vector $\underline{u}$ the input at time t. $\underline{x}_o$ is the state of the system at time t_o (initial conditions) and $\dot{\underline{x}}$ is the derivative of $\underline{x}$ with respect to time. $\underline{f}$ and $\underline{h}$ are vectors that describe the structure of the system. The main assumption to be made at this stage about the above system in order that meaningful results can be obtained is that there will be unique values of $\underline{x}(t)$ and $\underline{y}(t)$ for a given $\underline{u}(t)$ and $\underline{x}(t_o) = \underline{x}_o$ for all t such that $t > t_o$.

The purpose of any technique of system identification will then be, by operating on $\underline{u}(t)$ and $\underline{y}(t)$, the determination of $\underline{f}$ and $\underline{h}$. As said before it will be, in

general, not possible to obtain the solution to this problem. Therefore, the assumption of a known structure except for a set of unknown parameters is made and the system is expressed by the following:

$$\underline{\dot{x}} = \underline{f}(\underline{x},\underline{u},\underline{p},t) \qquad \underline{x}(t_o) = \underline{x}_o \tag{1.3}$$

$$\underline{y} = \underline{g}(\underline{x},\underline{u},\underline{p},t) \tag{1.4}$$

where the vector $\underline{p}$ represents the set of parameters to be determined.

In order that the identification of those parameters be of some value, it is further assumed that $\underline{p}$ is time invariant in the time interval of interest $t_o < t \leq t_1$, or at least that its time dependence is known meaning that once some $\underline{p}(\tau)$, $t_o < \tau < t_1$, is determined, $\underline{p}(t)$ will be known in the interval $[t_o, t_1]$.

It must be emphasized at this stage that the foregoing does not assume the uniqueness of that representation. Only the uniqueness of the input/output relationship is assumed. That is, there will be for a given $\underline{u}(t_o,t)$ and $\underline{y}(t)$, a unique $\underline{b}$ such that

$$\underline{y}(t) = \underline{b}(\underline{u}(t_o,t)\ ,\ t) \tag{1.5}$$

It will be possible to show the existence of a representation $\underline{x}'$, $\underline{f}'$, $\underline{h}'$ and $\underline{p}'$ such that

$$\underline{\dot{x}}' = \underline{f}'(\underline{x}',\underline{u},\underline{p}',t) \qquad \underline{x}'(t_o) = \underline{x}_o' \tag{1.6}$$

$$\underline{y} = \underline{h}'(\underline{x}'\underline{u},\underline{p}',t) \tag{1.7}$$

The problem is then redefined as the determination of a set of parameters $\underline{p}$ of some state representation of the system defined by $\underline{y}(t) = \underline{b}(\underline{u}(t_0,t), t)$.

Coming back to the case of an ocean vehicle, it will be shown that the resulting equations of motion may be regarded as a state representation of the vehicle dynamical behaviour and that the assumptions made so far are met by that mathematical model.

The foregoing assumes, apart from the unknown set of parameters $\underline{p}$, no other types of uncertainty. But, as said before, it is sometimes advantageous to select a simpler mathematical model for a given system and introduce the so-called system or environmental noise since it results from interactions between the system and the environment. Also consideration must be given to measurement noise. A model that takes into account those factors will be of the following form [2] :

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u},\underline{p},\underline{w},t) \qquad \underline{x}(t_0) = \underline{x}_0 \qquad (1.8)$$

$$\underline{y} = \underline{h}(\underline{x},\underline{u},\underline{p},\underline{v},t) \qquad (1.9)$$

where the vector $\underline{w}$ represents the environmental noise of the system[1] and the vector $\underline{v}$ the measurement noise.

[1] Errors in the structure of the system will come through as environmental noise.

In addition (case of time varying systems) the parameter vector $\underline{p}$ may have the following structure:

$$\dot{\underline{p}} = \underline{g}(\underline{p},t,\underline{w}') \qquad \underline{p}(t_o) = \underline{p}_o \qquad (1.10)$$

where $\underline{g}$ is known and $\underline{w}'$ is noise.

In order to simplify the above formulation the following assumptions are generally made:

1. The mathematical model is time invariant.

2. System structural uncertainty will appear as an apparent additive noise process.

3. Measurement noise may also be considered an additive noise process.

4. Output measurements are linear functions of the state of the system and are structurally independent of the input.

5. Parameters are coefficients in the model structure that are to be identified.

The system dynamical behaviour may be expressed as follows:

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u},\underline{p}) + \underline{w} \qquad (1.11)$$

$$\dot{\underline{p}} = o \qquad (1.12)$$

$$\underline{y} = \underline{H}\ \underline{x} + \underline{v} \qquad (1.13)$$

where $\underline{H}$ is the measurement matrix, often a diagonal matrix.

The purpose of the identification will be the determination of p.

In Chapter 2 the equations of motion for an ocean vehicle will be developed and it will be shown that indeed they constitute a state space representation of the ocean vehicle's dynamical behaviour. It will be shown also that they satisfy the foregoing assumptions.

In Chapter 3 the methods of predicting the ship model motion parameters will be outlined and values of those parameters for the mariner class hull form will be presented (Appendices 1, 2, 3).

In Chapter 4 the excitation from the environment will be described and its several components isolated.

In Chapter 5 the experimental procedure followed in order to obtain the data will be described.

In Chapter 6 the different methods of system identification will be discussed and those which seem more convenient will be used to identify the motion parameters of the mariner class hull form.

In Chapter 7 the results obtained in Chapter 6 will be discussed and in Chapter 8 the consequent conclusions and recommendations will be pointed out.

CHAPTER 2

EQUATION OF MOTION

In order to develop the equations of motion for an ocean vehicle, it is convenient first to define two rectangular right handed coordinate systems, one fixed in the surface of undisturbed water with the positive $x_3 = z_o$ axis pointing downward and the second moving with the vehicle, the origin of such coordinate system being arbitrary (but the coordinates (x_G, y_G, z_G) of the center of gravity of the vehicle in this system being known) pointing the positive $x_1 = x$ axis to the bow of the vehicle and the $x_3 = z$ pointing downward in the equilibrium position (see Figure 2.1)

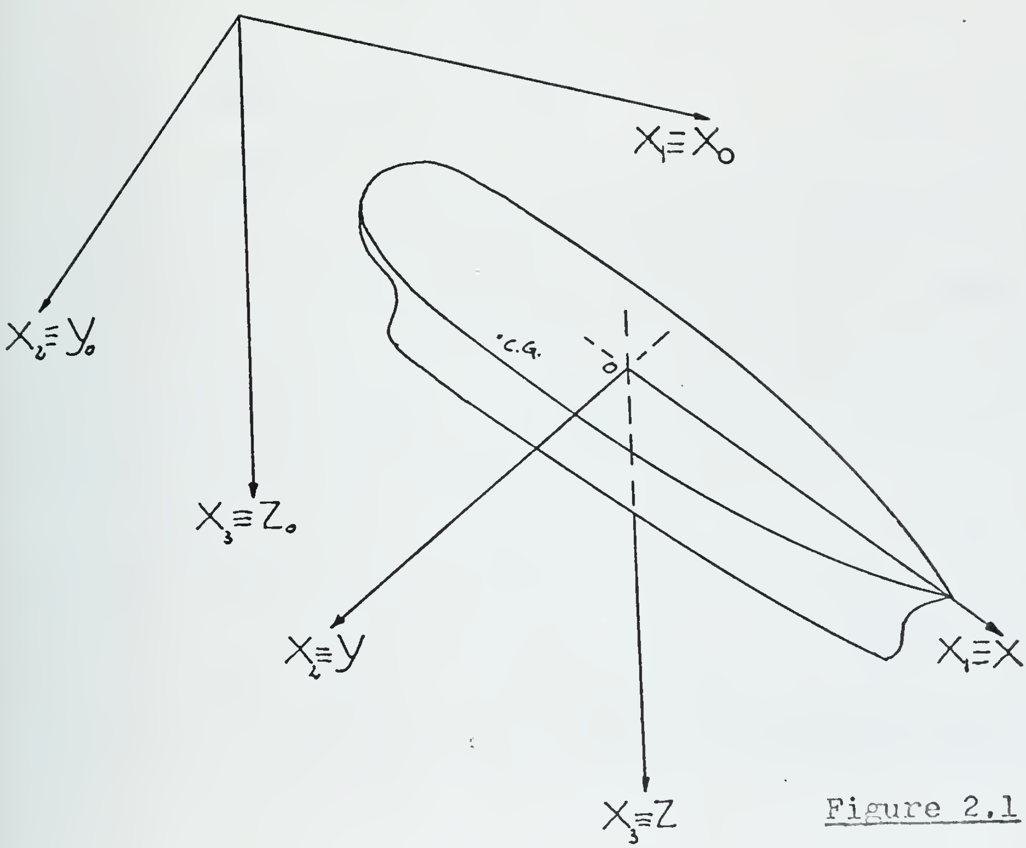

Figure 2.1

The orientation of the moving axis system with respect to the fixed axis system will be expressed by three angles (the "modified Euler angles") defined as follows, assuming that initially both axis systems coincide [3]:

ψ - angle of rotation of the moving axis system about the $z = z_0$ axis (yaw angle)

θ - rotation of the moving coordinate system already yawed about its y axis (pitch angle)

ϕ - rotation of the moving coordinate system (already yawed and pitched) about its x axis (roll angle).

Defining the rotation about the body axis as respectively S_4(about the body x_1 axis), S_5(about the body x_2 axis) and S_6(about the body x_3 axis) the relation between those rotations and the Euler angles will be [3]:

$$\begin{aligned} S_4 &= \phi\cos\theta\cos\psi - \theta\sin\psi \\ S_5 &= \phi\cos\theta\sin\psi + \theta\cos\psi \\ S_6 &= -\phi\sin\theta + \psi \end{aligned} \tag{2,1}$$

The motion of a rigid body in six degrees of freedom with respect to an axis system fixed in space, when the origin is at the center of gravity and the moving axes are the principal axes of inertia, may be represented by Newton's equations, namely:

$$\begin{aligned} \underline{F} &= \frac{d}{dt}\left(\underline{m}\underline{u}_G\right) \\ \underline{M} &= \frac{d}{dt}\left(\underline{I}\underline{\Omega}_G\right) \end{aligned} \tag{2,2}$$

where $\underline{m}$ is a diagonal matrix whose elements are equal to

the mass of the body, $\underline{u}_G$ is the translational velocity vector of the center of gravity, of components along the moving axes u_G, v_G and w_G, $\underline{F}$ is the total vector force acting on the body of components along the axes X, Y and Z, $\underline{I}$ is a diagonal matrix whose elements are the inertia, $\underline{\Omega}_G$ is the rotational velocity vector about those axes of components p_G, q_G and r_G and $\underline{M}$ is the total moment acting on the body of components K, M, and N. Neglecting the effects due to rotation of the earth in ocean vehicle dynamics Abkowitz [4] and Tufts [5] develop equations (2.2) respectively with fixed and varying mass and position of center of gravity for the case of an arbitrary origin, the first using vector analysis and the second Lagrange's equations. Therefore only the resulting equations will be presented here and for a body of fixed mass and inertia.

$$\begin{aligned} X &= m[\dot{u}+qw-rv-x_G(q^2+r^2)+y_G(pq-\dot{r})+z_G(pr+\dot{q})] \\ Y &= m[\dot{v}+ru-pw-y_G(r^2+p^2)+z_G(qr-\dot{p})+x_G(qp+\dot{r})] \\ Z &= m[\dot{w}+pv-qu-z_G(p^2+q^2)+x_G(rp-\dot{q})+y_G(rq+\dot{p})] \end{aligned} \qquad (2.3)$$

$$\begin{aligned} K &= I_{xx}\dot{p} + I_{xy}(\dot{q}-pr) + I_{xz}(\dot{r}+pq) + I_{yz}(q^2-r^2) + \\ &\quad (I_{zz}-I_{yy})qr - mz_G(\dot{v}-pw+ru) + my_G(\dot{w}-qu+pv) \\ M &= I_{yy}\dot{q} + I_{yz}(\dot{r}-qp) + I_{yx}(\dot{p}+qr) + I_{zx}(r^2-p^2) + \\ &\quad (I_{xx}-I_{zz})rp - mx_G(\dot{w}-qu+pv) + mz_G(\dot{u}-rv+qw) \end{aligned} \qquad (2.4)$$

$$N = I_{zz}\dot{r} + I_{zx}(\dot{p}-rq) + I_{zy}(\dot{q}+rp) + I_{xy}(p^2-q^2) + (I_{yy}-I_{xx})pq - my_G(\dot{u}-rv+qw) + mx_G(\dot{v}-pw+ru) \tag{2.4}$$

where unsubscripted velocities and accelerations refer to an arbitrary origin and x_G, y_G and z_G are the coordinates of the center of gravity.

Note that $\underline{F}$ and $\underline{M}$ will in general be functions of position, velocity and acceleration of the vehicle as well of properties of fluid, geometry of the body and of the inputs. For a body of given geometry and mass distribution and in a given fluid then

$$\begin{matrix} \underline{F} \\ \underline{M} \end{matrix} = \begin{matrix} F \\ M \end{matrix} \left[\underline{u}, \underline{\Omega}, \dot{\underline{u}}, \dot{\underline{\Omega}}, \underline{x}, \underline{s}, \underline{\text{input}}\right]$$

where
$$\underline{u} = [u, v, w]$$
$$\underline{\Omega} = [p, q, r] \tag{2.5}$$
$$\underline{x} = [x_o, y_o, z_o]$$
$$\underline{s} = [\psi, \theta, \phi]$$

Since the purpose of this investigation was the study of the applicability of system identification techniques to ocean vehicle dynamics and not the identification of parameters of a particular vehicle (parameters imbedded in $\underline{F}$ and $\underline{M}$) it was decided at this stage to simplify the formulation by restraining the model to one or at most two degrees of freedom. It will

be only considered heave (translation along the z axis) and roll (rotation about the x axis) with zero forward speed. To simplify the formulation even more y_G and z_G were made zero. Therefore, the equations of motion for this restrained mode ($y_G=z_G=\dot{u}=u=\dot{v}=v=\dot{q}=q=\dot{r}=r=x_o=y_o= =\theta=0$) will be

$$Z = m\dot{w}$$
$$K = I_{xx}\dot{p} \qquad (2.6)$$

and

$$Z = Z[\dot{w}, w, z_o, \dot{p}, p, \phi, \underline{\text{input}}]$$
$$K = K[\dot{w}, w, z_o, \dot{p}, p, \phi, \underline{\text{input}}] \qquad (2.7)$$

Looking at equations (2.6 and (2.7) one sees that they are almost in a state space form except for the fact that Z and K are functions of $\dot{w}$ and $\dot{p}$ and that

$$p = \frac{d}{dt}[s_4] = \frac{d}{dt}[\phi\cos\theta\ \cos\psi - \theta\sin\psi] \quad \text{from Eq. (2.1)}$$

$$\dot{z}_o = w\cos\phi + v\sin\phi \quad \text{see Fig. 2.2}$$

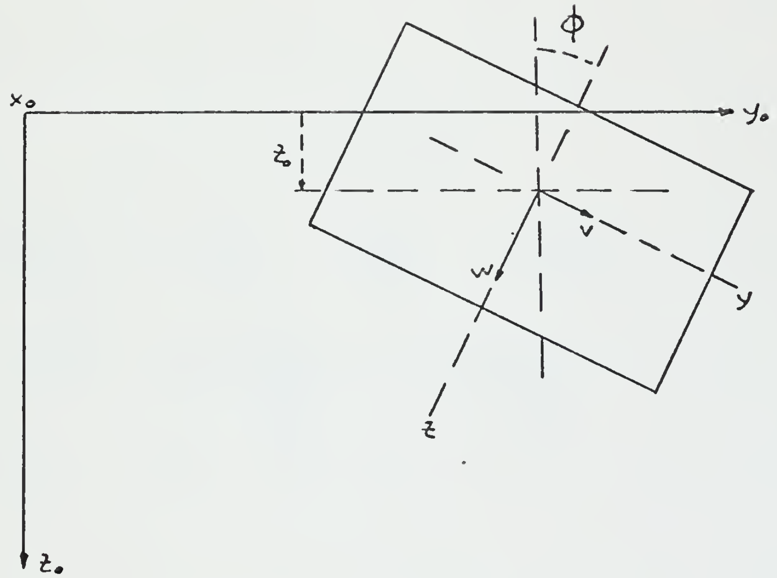

Figure 2.2

Since ψ, θ and v are zero, the above relations reduce to

$$\begin{aligned} \dot{\phi} &= p \\ \dot{z}_o &= w \cos\phi \end{aligned} \qquad (2.8)$$

Ignoring for the moment the presence of $\dot{w}$ and $\dot{p}$ as arguments of Z and K the complete set of equations in state space form would be:

$$\begin{aligned} \dot{w} &= \frac{1}{m} \; Z(\dot{w}, w, z_o, \dot{p}, p, \phi, \underline{\text{input}}) \\ \dot{z}_o &= w \cos\phi \\ \dot{p} &= \frac{1}{I_{xx}} \; K(\dot{w}, w, z_o, \dot{p}, p, \phi, \underline{\text{input}}) \\ \dot{\phi} &= p \end{aligned} \qquad (2.9)$$

In order to be able to proceed and apply the existing identification techniques to ocean vehicles (specifically surface ships), it is necessary to isolate $\dot{w}$ and that the dependence of Z and K on $\dot{w}$ and $\dot{p}$ is linear. Therefore, the next step will be to assume the existence of a Taylor's expansion about an equilibrium point, to be taken as $\dot{w}=w=\text{-----}=\phi=0$ of the form:

$$Z = Z(o) + \left.\frac{\delta Z}{\delta \dot{w}}\right|_o \dot{w} + \left.\frac{\delta Z}{\delta w}\right|_o w + \left.\frac{\delta Z}{\delta z_o}\right|_o z_o + \left.\frac{\delta Z}{\delta \dot{p}}\right|_o \dot{p} + \text{------} + \left.\frac{\delta Z}{\delta \, \underline{\text{inp}}}\right|_o \underline{\text{input}} + \frac{1}{2!} \left.\frac{\delta^2 Z}{\delta \dot{w}^2}\right|_o \dot{w}^2 + \frac{1}{2!} \left.\frac{\delta^2 Z}{\delta w^2}\right|_o w^2 + \text{-----} \qquad (2.10)$$

$$+ \frac{1}{2!}\left.\frac{\delta^2 Z}{\delta \dot{w}\delta w}\right|_o \dot{w}w + \frac{1}{2!}\left.\frac{\delta^2 Z}{\delta \dot{w}\delta z_o}\right|_o \dot{w}z_o + \frac{1}{2!}\left.\frac{\delta^2 Z}{\delta \dot{w}\delta \dot{p}}\right|_o \dot{w}\dot{p} + ---- + \qquad (2.10)$$

and in the same way for K.

It is known from potential theory [5] that for a body moving in an infinite fluid away from the surface there are no dependence of force or moments on products of accelerations with velocities or accelerations. It will be necessary to accept this assumption for surface vehicles without any other justification, and neglect also all other terms where acceleration is multiplying any other variable.

With the vehicle in equilibrium also $Z(o) = 0$.

Further simplification can be achieved if we consider each element of the expansion, which will be considered in the next chapter. It will be enough to say for the moment that due to port and starboard symmetry (both geometrically and inertial) the vertical motion of the vehicle does not produce any rolling moment, at least to first order, and therefore those terms can be dropped from the equations further simplifying them [4]. Similarly, and to first order only, rolling motions will not produce any vertical force or, in other words:

$$\left.\frac{\delta Z}{\delta \dot{p}}\right|_o = \left.\frac{\delta Z}{\delta p}\right|_o = \left.\frac{\delta Z}{\delta \phi}\right|_o = \left.\frac{\delta K}{\delta \dot{w}}\right|_o = \left.\frac{\delta K}{\delta w}\right|_o = \left.\frac{\delta K}{\delta z_o}\right|_o = 0 \qquad (2.11)$$

Also the square terms $\frac{\delta^2 Z}{\delta w^2}$ and $\frac{\delta^2 K}{\delta p^2}$ will be nonexistent. Instead, consideration must be given to frictional and vortical effects which are of the form $c_1 w|w|$ and $c_2 p|p|$. Although these terms are not analytic, they will be included so that the model represents a closer approximation to the vehicle physical behaviour.

The form of the equations will then be:

$$\dot{w} = \frac{1}{-Z_{\dot{w}}+m}\left[Z_w w + Z_{w|w|} w|w| + Z_{z_o} z_o + \ldots + \frac{1}{(m+n)!} Z_{x_i^n x_j^m} (x_i)^n (x_j)^m + \text{+input}\right]$$

$$\dot{z}_o = w \cos\phi \qquad (2.12)$$

$$\dot{p} = \frac{1}{-K_{\dot{p}}+I_{xx}}\left[K_p p + K_{p|p|} p|p| + \ldots \frac{1}{(m+n)!} K_{x_j^n x_i^m} (x_j)^n (x_i)^m + \text{input}\right]$$

$$\dot{\phi} = p$$

where $Z_{x_j} = \frac{\delta Z}{\delta x_j}$ and $K_{x_i} = \frac{\delta K}{\delta x_i}$

In the next chapter each term in the above equations which is of first order, quadratic or at most cubic will be discussed in relation to its nature, order of magnitude and methods of predicting its value.

In summary, from Newton's equations and for a case of restricted motions, assuming the existence of a Taylor series for the force and moment and linear dependence of

force and moment on acceleration it was possible to obtain a set of equations in a state space representation to which some system identification techniques may be applicable.

CHAPTER 3

MOTION PARAMETERS

For convenience of presentation, the discussion on the motion parameters will be subdivided into three main groups, namely:

i) motion parameters resultant of differentiation of force or moment on the body with respect to position (z_0,ϕ) only (hydrostatic coefficients)

ii) linear terms resultant from differentiation of force or moment on the body with respect to acceleration and velocity (added mass or added moment of inertia and damping coefficients)

iii) non-linear hydrodynamic terms

3.1- Hydrostatic Coefficients

These coefficients are dependent only on the geometry and mass distribution of the body, for example, shape and relative position of the metacenter, center of buoyancy and center of gravity, for the restoring moments.

The coefficients are calculated based on the lines plan of the hull form.

With the ship in the upright position, the buoyant force B (z_0,0) is calculated for each waterline (each

selected value of z_o). Then at each waterline an angle of roll is simulated in the body plan, being the point of rotation the assumed position of the center of gravity, and the resultant buoyant force $B(z_o, \phi)$ and the righting arm $GZ(z_o,\phi)$ are calculated (for details see, for example, "Principles of Naval Architecture" edited by J. P. Comstock, Chapter 2). The righting moment ($-K(z_o,\phi)$) is then given by the product of $B(z_o,\phi) \cdot GZ(z_o,\phi)$. The vertical force, positive downwards, is the difference between the weight of the body and the buoyant force, or in other words, considering the component of the force along the body x_3 axis. We have:

$$\begin{aligned} Z(z_o,\phi) &= (W - B(z_o,\phi)) \cos \phi \\ K(z_o,\phi) &= -B(z_o,\phi) \cdot GZ(z_o,\phi) \end{aligned} \qquad (3.1)$$

where W is the weight of the vehicle.

The values of $Z(z_o,\phi)$ and $K(z_o,\phi)$ are usually presented tabulated or in graphical form (see Appendix 1). In order to get the hydrostatic coefficients, derivatives of (3.1), it is necessary to obtain a mathematical representation of those surfaces, for example, a polynomial fitting by the method of least squares. One such scheme was developed by Kerwin [6] and implemented by Parissis [7] using Legendre Polynomials to represent ship forms which is equally applicable to (3.1).

If one is interested in using in the state equations just the linear terms such a procedure is not necessary. It is easily seen that

$$\left.\frac{\delta Z(z_o,\phi)}{\delta \phi}\right|_{z_o=\phi=0} = \left.\frac{\delta K(z_o,\phi)}{\delta z_o}\right|_{z_o=\phi=0} = 0 \qquad (3.2)$$

due to port-starboard symmetry of the hull form and that

$$\left.\frac{\delta Z(z_o,\phi)}{\delta z_o}\right|_{z_o=\phi=0} = -\rho g A_{WL}$$

$$\left.\frac{\delta K(z_o,\phi)}{\delta}\right|_{z_o=\phi=0} = -W\cdot GM \qquad (3.3)$$

where A_{WL} represents the area of the waterplane at the design waterline ($z_o=0$) and GM is the metacentric height (distance from the metacenter to the center of gravity).

Due also to port and starboard symmetry, in the non-linear formulation the term $\left.\frac{1}{2!}\cdot\frac{\delta^2 K(z_o,\ )}{\delta\ 2}\right|_{z_o=\phi=0}$ is equal to zero. Therefore in this case the cubic term will be maintained in the equations.

3.2 - Added Mass, Added Moment of Inertia dnd Damping Coefficients

From the eight terms that form this subgroup four will be zero again due to symmetry of the vehicle. Those will be the cross coupling terms

$$-Z_{\dot{p}} = Z_p = -K_{\dot{w}} = -K_w = 0 \qquad (3.4)$$

and the terms to be retained:

$$-Z_{\dot{w}}$$
$$-Z_{w} \qquad (3.5)$$
$$-K_{\dot{p}}$$
$$-K_{p}$$

So it is seen that when only linear terms are considered the equations of heave and roll are completely decoupled and may be studied independently. If the six degrees of freedom were considered, the existence of port and starboard symmetry would cause, when retaining only the linear terms in the equations of motion, the decoupling of the equations into two groups [3,4]: coupled surge (translation along x_1 axis), heave (translation along the x_3) and pitch (rotation about the x_2 axis) - the so-called motions in the vertical plane, and coupled sway (translation along the x_2 axis), roll (rotation about the x_1 axis) and yaw (rotation about the x_3 axis) in the horizontal plane.

Added mass and damping coefficients are in general not known except for some special types of idealized forms and when the vehicles are excited into some special types of motions. One such type that have been studied quite extensively, especially in the domain of surface vehicles or vehicles near the surface, is that of a

vehicle excited by waves into an oscillatory motion. The path usually followed [3,8,9] is to assume the fluid to be inviscid, incompressible and irrotational, therefore assuring the existence of a velocity potential that satisfies both the Laplace equation and the Bernoulli equation. Considering the motion harmonic, the solution is sought as the product Re $[\varphi(x_1,x_2,x_3)e^{-i\omega t}]$ where ω is the frequency of oscillation. Considering the linearized problem, $\varphi(x_1,x_2,x_3)$ is assumed to be the sum of eight other potentials being the total motion the superposition of each due to each of the potentials: two of them due to the action of the incident wave plus diffracted wave on a fixed vehicle and six due to forced oscillations of the vehicle on calm water. The fluid is assumed in all cases to extend to $\pm\infty$ in the x_1 and x_2 directions and between $x_3=0$ and $+\infty$ or between $x_3=0$ and $x_3=h$ for the finite depth case. The linearized boundary conditions will be:

a) On the surface $x_3=0$, the velocity of the fluid $\frac{\delta\Phi}{\delta x_3} = \frac{\delta\zeta}{\delta t}$ where ζ is the surface elevation, for all the potentials and is equal to $\frac{-\omega^2}{g}\varphi_j$ using the linearized Bernoulli equation.

b) On the body for the incident and diffracted

potentials $\frac{\delta\varphi}{\delta n} = 0 = \frac{\delta}{\delta n}(\varphi_w + \varphi_d)$ where n is the unit normal.

For the six other potentials the normal component of the velocity will equal the velocity of the body in that direction or $R\left[\frac{\delta\varphi_j}{\delta n} \quad e^{-i\omega t}\right] = f_j\dot{s}_j \Big|$ or being

$\dot{s}_j = R\left[-i\omega s_{ja} e^{-i\omega t}\right]$ (s_{ja} is the amplitude of the oscillation in the j direction)

$$\frac{\delta\varphi_j}{\delta n} = -i\omega f_j s_{ja} \qquad (3.6)$$

where f_j (j = 1,....6) are the generalized direction cosines of the body surface

$$\begin{aligned} f_j \ (j=1,2,3) &= \cos\,(n,x_j) \\ f_4 &= x_2 f_3 - x_3 f_2 \\ f_5 &= x_3 f_1 - x_1 f_3 \\ f_6 &= x_1 f_2 - x_2 f_1 \end{aligned}$$

c) At

$$\begin{aligned} x_3 &= +\infty & \nabla\Phi &= 0 \\ x_3 &= +h & \frac{\delta\Phi}{\delta n} &= 0 \end{aligned} \qquad (3.7)$$

d) At $r = (x_1^2 + x_2^2)^{\frac{1}{2}}$ the radiation condition must be satisfied for the six potentials and for diffraction potential or:

$$\lim_{r\to\infty} r^{\frac{1}{2}}\left[\frac{\delta\varphi_j}{\delta r} + \frac{i\omega^2}{g}\varphi_j\right] = 0 \qquad j = 1\ldots 6,d \quad (3.8)$$

Also from Bernoulli equation the hydrodynamic pressure will be

$$p = -\rho\frac{\delta\Phi}{\delta t} = \mathrm{Re}\left[i\omega\rho\varphi e^{-i\omega t}\right] = \mathrm{Re}\left[i\omega\rho\left[\varphi_w + \varphi_d + \sum_{j=1}^{6}\varphi_j\right]e^{-i\omega t}\right] \tag{3.9}$$

The hydrodynamic force in the k direction will be

$$F_k = -\iint_{S_o} p f_k ds = \mathrm{Re}\left[-i\omega\rho e^{-i\omega t}\iint_{S_o}\left[\varphi_w + \varphi_d + \sum_{j=1}^{6}\varphi_j\right]f_k ds\right] \tag{3.10}$$

The total hydrodynamic force on the body in a given direction k was defined to be in Chapter 2 for motions in mode j: $F_{kj} = \frac{\delta F_{kj}}{\delta s_j}s_j + \frac{\delta F_{kj}}{\delta \dot{s}_j}\dot{s}_j$

being
$$\dot{s}_j = \mathrm{Re}\left[-i\omega s_{ja}e^{-i\omega t}\right]$$
$$s_j = \mathrm{Re}\left[-\omega^2 s_{ja}e^{-i\omega t}\right]$$

and changing the nomenclature to

$$\frac{\delta F_{kj}}{\delta s_j} = -A_{kj}, \quad \frac{\delta F_{kj}}{\delta \dot{s}_j} = -B_{kj}$$

$$F_{kj} = \mathrm{Re}\left[(\omega^2 A_{kj} + i\,\omega\, B_{kj})s_{ja}\, e^{-i\omega t}\right] \tag{3.11}$$

and the total force in the k direction was

$$F_k = \sum_{j=1}^{6}F_{kj} + F_{kw}$$

F_{kw} being the excitation in the k direction (the hydrostatic force was excluded from the foregoing expression as it was from the Bernoulli equation (3.9)).

Comparing the foregoing expression with (3.10), the following is obtained:

$$F_{wk} = Re\left[-i\omega\rho e^{-i\omega t}\iint_{S_o} (\varphi_w + \varphi_d) f_k ds\right] \tag{3.12}$$

$$(\omega^2 A_{kj} + i\omega B_{kj}) s_{ja} = -i\omega\rho\iint_{S_o} \varphi_j f_k ds \tag{3.13}$$

and therefore we obtain

$$A_{kj} = \frac{\rho}{s_{ja}\omega} \iint_{S_o} Im\left[\varphi_j\right] f_k ds$$

$$B_{kj} = -\frac{\rho}{s_{ja}} \iint_{S_o} Re\left[\varphi_j\right] f_k ds \tag{3.14}$$

that would give the value of the hydrodynamic coefficients if the velocity potential was known. Since that is not the case, it is not possible to use the above equations. Instead one looks for possible relationships between the hydrodynamic coefficients to get a better insight on the solution of the problem.

For example using the second form of Green's theorem to a velocity potential φ_j and to another velocity potential φ_k with $k \neq j$ we obtain since both potentials satisfy Laplace's equation: [3]

$$\begin{aligned} A_{kj} &= A_{jk} \\ B_{kj} &= B_{kj} \end{aligned} \tag{3.15}$$

Also from [9] and [10] the following expression is obtained relating added mass and damping coefficients:

$$A_{kj}(\omega) - A_{kj}(\infty) = \frac{2}{\pi} \int_0^\infty [B_{kj}(\xi) - B_{kj}(\infty)] \frac{d\xi}{\xi^2 - \omega^2} \qquad (3.16)$$

$$B_{kj}(\omega) - B_{kj}(\infty) = \frac{2}{\pi} \int_0^\infty [A_{kj}(\xi) - A_{kj}(\infty)] \frac{\xi^2 d\xi}{\xi^2 - \omega^2}$$

Maybe the most important relations are the so-called Haskind relations [11] based also on the second form of Green's theorem when applied to equation (3.12) after the expression for f_k in (3.6) or:

$$f_k = -\frac{\delta \varphi_k}{\delta n} \quad \frac{1}{i\omega s_{ka}} \qquad (3.6)$$

is substituted in (3.12):

$$F_{wk} = \mathrm{Re}\left[\frac{\rho}{s_{ka}} e^{-i\omega t} \iint_{s_o} (\varphi_w + \varphi_d) \frac{\delta \varphi_k}{\delta n} ds\right] \qquad (3.12)$$

Green's theorem

$$\iiint_R (\varphi \nabla^2 \psi - \psi \nabla^2 \varphi)\, dr = \oiint_{s_o} \left(\varphi \frac{\delta \psi}{\delta n} - \psi \frac{\delta \varphi}{\delta n}\right) ds \qquad (3.13)$$

for $\varphi = \varphi_d$

$\psi = \varphi_k$

Since both potentials satisfy Laplace's equation the left hand side is zero and therefore:

$$\iint_{s_o} \varphi_d \cdot \frac{\delta \varphi_k}{\delta n} ds = \iint_{s_o} \varphi_k \frac{\delta \varphi_d}{\delta n} ds \qquad (3.14)$$

and from the boundary condition on the body

$$\frac{\delta \varphi_d}{\delta n} = -\frac{\delta \varphi_w}{\delta n} \qquad (3.15)$$

The following will result for the excitation force:

$$F_{wk} = Re\left[\frac{\rho}{s_{ka}} e^{-i\omega t} \iint_{S_o} \left[\varphi_w \frac{\delta \varphi_k}{\delta n} - \varphi_k \frac{\delta \varphi_w}{\delta n}\right] ds\right] \qquad (3.16)$$

eliminating the diffraction potential.

Following [11] for 2-dimensional bodies (under beam seas) the following relations are obtained, based on (3.16):

$$B_{kk} = \frac{\omega}{\rho g^2 \zeta_a^2} \left| F_{ka} \right|^2$$

and therefore (3.17)

$$F_{ka} = \zeta_a \left[\frac{g^2}{\omega} B_{kk}\right]^{\frac{1}{2}} \qquad k = 2,3,4$$

where ζ_a is the amplitude of the incoming waves and F_{ka} is the amplitude of the exciting force. Being the damping coefficient calculated from the energy radiated at infinity it's possible to calculate the magnitude of the exciting force. However, it's phase is not determined.

From the foregoing it is seen that the analytic solution for the general case of a three dimensional body does not exist. Only the cases of the spheres and ellipsoids have been solved [12]. It would be possible to solve a problem numerically by such a procedure as the finite element techrique [3], but nothing is available yet.

In order to get information for practical applications it is necessary then to work into two dimensional problems. The strip theory for ship motions, introduced by Korvin-Kroukovsky [13, 14, 15] makes use of two dimensional solutions for each transverse section of the body

determining the characteristics on the flow and consequently the local hydrodynamic coefficients and then the results are integrated over the length of the body to obtain the hydrodynamic coefficients for the whole vehicle, applying then a correction factor to take into account the existence of a three dimensional flow.

The solution in two dimensions has been obtained for the three possible modes of motion: sway, heave and roll. One way to obtain the solution is, using conformal mapping to transform the cross section of arbitrary shape of a two dimensional cylinder into the unit circle, to assume the velocity potential to be a sum of multipole potentials whose coefficients are adjusted to satisfy the boundary conditions on the cylinder [16]. For example for heave the multipole expansion will be formed by the sum of a source potential at the origin plus the multipole potentials while for sway and roll instead of a source there will be a dipole since the heave potential is symmetrical and the other two asymmetrical.

The conformal transformation will be of the form [8]

$$\frac{z}{a} = \zeta + \sum_{n=0}^{N} a_{2n+1} \cdot \zeta^{-(2n+1)} \qquad (3.18)$$

which maps an arbitrary section in the z plane into the unit circle in the ζ plane. From the section offsets it is necessary to find the coefficients a_{n+1}. A method of determining those coefficients was developed by Bermejo [17] using Theodorsen's method.

Another way of solving the two dimensional problem is to represent the velocity potential by a distribution of sources on the contour of the section [18]. The intensity of the sources is determined applying the kinematic boundary condition to the underwater portion of the section. Once the potential is obtained the pressure is calculated using equation (3.9) - linearized Bernoulli equation - and the force is obtained by integration from where the added mass and damping coefficient are calculated (Eq. 3.9 to 3.12).

Based on this method a computer program was developed [19] which performs the above calculations and was used in this investigation for the computation of the initial conditions of those motion parameters. Listings of those programs are presented in Appendix 2.

Still another way of calculating hydrodynamic coefficients is to use approximate formulas, based some of them on the same principles as exposed above,

and some in model experiments and full scale information. Some of those formulas are presented by Blagoveshchensky [20] and were used here to calculate A_{33}, A_{44}, B_{33} and B_{44} and the results compared with those obtained with the computer program mentioned, believed to be more exact. Those results were also compared with the ones obtained by Hines and Gies [21] in experiments with another model of the same hull form. Those results are presented in Appendix 3.

3.3 - Non-linear Hydrodynamic Coefficients

In this category are the terms of second order in velocity and cross terms in velocity and position.

Information on those types of parameters is practically nonexistent. Only two references to frictional damping in roll were found [20, 22]. Since that is the case where frictional and separation resistance has a bigger effect when compared with the other modes of motion, the expressions suggested by Blagoveshchensky was used since it was directly applicable to ocean vehicles.

The expression given by reference [22] relates the friction coefficient of oscillating cyclinders to an equivalent Reynolds number:

$$c_f = 1.328 \left[\frac{3.22 \; r^2 \; \dot{\theta}_m^2}{T \nu} \right]^{-.5} \qquad (3.19)$$

where r - radius of the cylinder

θ_m - mean amplitude of the oscillation

T - period of oscillation

ν - kinematic viscosity

This formula is the same as blasius solution for C_f in laminar flow if the Reynolds number is taken as

$$Rey = \frac{3.22\, r^2\, \theta_m^{\,2}}{T\nu} \tag{3.20}$$

To obtain the drag coefficient for heaving oscillations a similar method may be used, although with rougher approximations since the vehicle is not a two dimensional body and the sections do not resemble circles.

Here a "strip theory" approach was used. Results were sought for each station and integrated over the length of the body.

For each station it is assumed that the resistance is due, when moving into the water, entirely to skin friction and when moving away from the water, entirely to separation. Therefore for the first case the diameter of the equivalent cylinder will be $d = l/\pi$ where l is the length of the underwater part of the contour of the section. For the second case the diameter of the equivalent cylinder will be equal to the beam of the section. The velocity used to calculate the

Reynolds number will be the "root mean square" value based on the tests performed. Values of skin-friction coefficient and form-drag coefficients are obtained from reference [23]. Results of this procedure together with the results of the coefficients as calculated by the method indicated in reference [20] are presented in Appendix 3.

With respect to the other coefficients, the assumption may be made that they are indeed very small and consequently the assigned value to be used in the identification procedure is zero.

Summarizing in this chapter the ship motion parameters were considered, their nature, importance and the available methods of prediction outlined. The results obtained for the chosen hull form are presented in Appendix 1, 2 and 3.

In the next chapter the exciting force and moment will be considered thus completing the description of the mathematical model.

CHAPTER 4

EXCITATION FROM THE ENVIRONMENT

From all possible types of excitation, only forces and moments due to the action of two dimensional plane progressive waves are considered because (1) they are well defined in mathematical terms and (2) easy to generate in a towing tank experiment.

The expression for the velocity potential is obtained following a path similar to the one followed in Chapter 3 for the calculation of relations between the linear hydrodynamic coefficients, namely that the velocity potential satisfies both Laplace and Bernoulli equations. Taking the case of an infinite fluid in all directions the following expressions hold [15, 23]:

$$\nabla^2 \Phi_w = 0$$

B.C.: $$\left.\frac{d(\zeta - x_3) = 0}{dt}\right|_{x_3=\zeta} \qquad \left.\frac{\delta\zeta}{\delta t} \approx \frac{\delta\ w}{\delta x_3}\right|_{x_3\approx 0}$$

$$p - p_{a+m} = 0 = -\left.\left(\frac{\delta\Phi_w}{\delta t} + \frac{1}{2}(\nabla\Phi_w)^2 - g\zeta\right)\right|_{x_3=\zeta} \qquad (4.1)$$

$$\approx \left.\frac{1}{g}\,\frac{\delta\Phi_w}{\delta t}\right|_{x_3=0}$$

$$\nabla\Phi_W = 0 \qquad x_3 \to \infty$$
$$\Phi_W = \text{finite as} \quad r = \sqrt{x_1^2 + x_2^2} \to \infty \tag{4.1}$$

where ζ is the surface displacement.

The solution is sought by separation of variables and assuming a harmonic dependence on time, or in other words, and for two dimensional behaviour:

$$\Phi_W(x_1,x_2,x_3,t) = \mathrm{Re}[\varphi_2(x_2)\cdot\varphi_3(x_3)\cdot e^{-i\omega t}] \tag{4.2}$$

Plugging this solution back into Laplace's equation we obtain

$$\frac{\varphi_2''(x_2)}{\varphi_2(x_2)} = -\frac{\varphi_3''(x_3)}{\varphi_3(x_3)} = -k^2 \tag{4.3}$$

To satisfy the last boundary condition k^2 must be positive being the solution for $\varphi_2(x_2)$ and $\varphi_3(x_3)$:

$$\varphi_2(x_2) = C_1 e^{ikx_2} + C_2 e^{-ikx_2}$$

$$\varphi_3(x_3) = D_1 e^{kx_3} + D_2 e^{-kx_3} \tag{4.4}$$

and to satisfy $\nabla\Phi_W = 0 \quad x_3 \to \infty \quad D_1 = 0$

$$\varphi_3(x_3) = D_2 e^{-kx_3}$$

and therefore

$$\Phi_W(x_2,x_3,t) = \mathrm{Re}[e^{-kx_3}(C_1 e^{i(kx_2-\omega t)} + C_2 e^{-i(kx_2+\omega t)})] \tag{4.5}$$

The solution with $C_2 = 0$ corresponds to waves progressing in the positive x_2 direction and with $C_1 = 0$ to waves progressing in the negative x_2 direction.

Assuming the later case and the existence of an arbitrary phase angle δ to be determined later from the initial conditions:

$$\Phi_w(x_2, x_3, t) = \mathrm{Re}\ C\ e^{-[kx_3 + i(kx_2 + \omega t + \delta)]} \qquad (4.6)$$

Combining the two boundary conditions on the surface $(x_3 \approx 0)$

$$\left.\begin{array}{l} \frac{1}{g} \frac{\delta^2 \Phi_w}{\delta t^2} = \frac{\delta w}{\delta x_3} \\ \frac{1}{g}\ \omega^2 = k \end{array}\right| x_3 = 0 \qquad (4.7)$$

which determines the value of k, the wave number.

Using $\left. \zeta = \frac{1}{g} \frac{\delta \Phi_w}{\delta t} \right|_{x_3 = 0}$ 4.1

$$= \frac{1}{g}\ \mathrm{Re}\left[-i\omega C\ e^{-i(kx_2 + \omega t + \delta)} \right]$$

$$= \frac{\omega C}{g} \sin(kx_2 + \omega t + \delta)$$

and for the form

$$\zeta = \zeta_a \sin(kx_2 + \omega t + \delta) \qquad (4.8)$$

where ζ_a is the amplitude of the surface displacement

$$C = \frac{g\zeta_a}{\omega} \qquad (4.9)$$

and

$$\Phi_w(x_2,x_3,t)=Re\left[\frac{g\,a}{\omega}\quad e^{-[kx_3+i(kx_2+\omega t+\delta)]}\right] \qquad (4.10)$$

from the Haskind relations, Eq. (3.16)

$$F_{wj}=Re\left[\frac{\rho}{s_{ja}}\; e^{-i\omega t}\iint_{S_o}[\,\varphi_w\,\frac{\delta\varphi_j}{\delta n}-\varphi_j\,\frac{\delta\varphi_w}{\delta n}\,]\,ds\right]$$

where $\varphi_w(x_1,x_2,x_3)=\ \Phi_w(x_1,x_2,x_3,t)\ e^{\ i\omega t}$

The first term in the integral represents the force exerted by the pressure in the wave without taking account of the disturbance by the presence of the ship and is the only excitation accounted for by the Froude-Kriloff hypothesis. The second term is the contribution of the diffraction of the waves from the body.

To compute the first term in the exciting force (the Froude-Kriloff force) first replace

$$\frac{\delta\varphi_j}{\delta n}=-\quad i\omega f_j s_{ja} \qquad (3.6)$$

$$(F_{wj})_{fk}=Re\left[-i\omega\rho\iint_{S_o}\quad\frac{g\,a}{\omega}\quad e^{-[kx_3+i(kx_2+\ t+\delta)]}\,f_j\,ds\right] \qquad (4.11)$$

$$=-\rho g a\iint_{S_o}\quad e^{-kx_3}\sin(kx_2+\omega t+\delta)\,f_j\,ds$$

The second term (diffraction force) is computed by noting that

$$\frac{\delta\varphi_w}{\delta n}=\ -\omega a\ e^{-[kx_3+i(kx_2+\delta)]}(f_2+if_3) \qquad (4.12)$$

$$(F_{wj})_d=Re\left[\frac{\rho\omega}{s_{ja}}\quad e^{-i\omega t}\iint_{S_o}\quad\varphi_j\ e^{-[kx_3+i(kx_2+\delta)]}\,(f_2+if_3)\,ds\right] \qquad (4.13)$$

and

$$F_{wj} = (F_{wj})_{fk} + (F_{wj})_{d} \tag{4.14}$$

Therefore to calculate the total excitation force the results from the sub-problem of forced oscillations of the body in calm water must be known, namely $\varphi_j(x_1, x_2, x_3)$. What was said about the way of calculating added mass and damping coefficients is applicable here. Besides it is possible to calculate this second term in the exciting force at the same time as the added mass and damping coefficients are calculated, since the potential is also obtained in the mode of interest. Or, in other words, we obtain for each station

$$\left[(F_{wj})_d\right]_{2dim.} = \mathrm{Re}\left[\frac{\rho\omega}{s_{ja}} e^{-i\omega t} \int_{C_o} {}_j e^{-[kx_3+i(kx_2+\delta)]} (f_2+if_3)\,dl\right] \tag{4.15}$$

where C_o is the contour of the station and dl is the length element along that contour.

These two dimensional results for each station are then integrated over the length of the vehicle.

Although this method seems to be the most suited for the computation of the exciting forces, that was not used in this investigation due to the fact that the computation of equation (4.15) was not included in the computer scheme used in the calculation of sectional added mass and damping coefficients. Instead, the

procedure presented by Blagoveshchensky [20] was used since, although less correct, rendered the computations much easier to perform.

First it is assumed that $\frac{\zeta_a}{\lambda} \ll 1$, $B/\lambda \ll 1$, $T/\lambda \ll 1$ where ζ_a is the wave amplitude, λ is the wave length $\lambda = 2\pi/k$, B and T are the beam and the draft of the vehicle.

Next, it is assumed that the principal part of the excitation is due to change in buoyancy due to the presence of the wave. Then some multiplicative correction coefficients are calculated to account for the curvature of wave (finite beam correction) and for the dependence of the hydrodynamic pressure on x_3 (finite draft correction). Each correction is calculated separately and assumed independent from each other (for example, when calculating the beam correction, $T/\lambda \ll 1$ and vice versa).

Under these assumptions the linear equations for heave and roll will take the form:

a) heave

$$\dot{w} = \frac{Z_w}{-Z_{\dot{w}}'+m} \cdot w + \frac{Z_{z_o}}{-Z_{\dot{w}}'+m} \cdot (z_o - \zeta_c) \qquad (4.16)$$

$$\dot{z}_o = w$$

where ζ_c is the surface displacement corrected for finite beam and draft effects

$$\zeta_c = \zeta \cdot C_{zb} \cdot C_{zt}$$
$$= {}_a \sin (ky + t + \delta)$$

b) roll

$$\dot{p} = \frac{K_p}{-K_{\dot{p}}+I_{xx}} p + \frac{[K_\phi - W.BG](\phi - \zeta^y_c)}{-K_{\dot{p}}+I_{xx}} + \frac{W.BG}{-K_{\dot{p}}+I_{xx}} \phi \qquad (4.17)$$

$$\dot{\phi} = p$$

where ζ^y_c is the wave slope corrected for finite beam and draft effects

$$\zeta^y_c = \zeta_y \cdot C_{\phi b} \cdot C_{\phi t}$$

$$\zeta_y = \frac{\delta\zeta}{\delta y} = k\zeta_a \cos(ky + \omega t + \delta)$$

C_{zb}, C_{zt}; $C_{\phi b}$, $C_{\phi t}$ are the multiplicative corrections given by [20]

$$C_{zb} = 1 - 1.73\, C_w\, (B/\lambda)^2$$
$$C_{zt} = 1 - C_m(kT) + \frac{C_m}{2(2-C_m)}(kT)^2 - \frac{C_m}{6(3-2C_m)}(kT)^3 \qquad (4.18)$$
$$C_{\phi b} = 1 - C_w\, (B/\lambda)^2$$
$$C_{\phi t} = 1 - \frac{6\, C_m^3}{(1+C_m)(1+2C_m)}(kT) + \frac{1.5\, C_m^3}{(2-C_m)(2+C_m)}(kT)^2 - \frac{C_m^3}{3(3-2C_m)(3-C_m)}(kT)^3$$

where

C_w is the waterline coefficient $C_w = A_{WL}/BL$

C_m is the midship section coefficient $C_m = A_{ST}/BT$

k is the wave number.

The details of the calculation of these correction coefficients may be found in Reference [20] paragraphs 27 to 31.

In Equation 4.17 the term $\dfrac{W.BG}{-k_{\dot{p}} + I_{xx}}$ represents the contribution due to the inclination of the ship when $\phi = \xi_c^y$ since the weight of the vehicle and its buoyancy will not be in the same vertical thus causing a heeling moment $W.BG \sin\phi$ being $W.BG.\phi$ the first term in the Taylor's expansion. W is the weight of the vehicle and BG is the distance from the center of gravity to the center of buoyancy.

Although the expressions above for the exciting force and moment are not as correct as Equations 4.11, 4.13 and 4.14, the simplicity of their calculation decided their introduction in the equations of motion.

When considering the non-linearities, if the exciting force can still be approximated as expressed in Equation 4.16 and 4.17, the following terms should be added to each pair of equations.

a) Heave

$$\frac{1/2\ Z_{w^2}}{-Z_{\dot{w}} + m} \cdot w|w| \quad \text{and} \quad \frac{1/2\ Z_{zo^2}}{-Z_{\dot{w}} + m} \cdot (z_o - \xi_c)^2 \qquad (4,19)$$

where the first term represents the frictional and form drag and the other term is the second term in the

polynomial fit of the $Z\,(z_o,\phi)\Big|_{\phi=0}$

b) roll

$$\frac{1/2\ k_{p^2}}{-k_{\dot{p}}+I_{xx}} \cdot p|p| \quad , \quad \frac{1/3!\,(k_{\phi^3} + W.BG)}{-k_{\dot{p}}+I_{xx}} \ (\phi - \xi_y)^3 \ \text{and} \tag{4,20}$$

$$\frac{1/3!\,W.BG}{-k_{\dot{p}}+I_{xx}} \cdot \phi^3$$

Where the first term is again the contribution of frictional and form drag, the second term is the contribution from the polynomial fit of the curves of stability and the last is the following term in the expansion of GB sin ϕ.

In the case above, the validity of the expression for the excitation is even more questionable due to the small orders of magnitude involved in the identification. One way of obtaining a better approximation is to consider the effects of the presence of the wave in the damping terms as is done for the case of roll by Tamya [22].

Those terms would then look like

a) heave

$$\frac{Z_w}{-Z_{\dot{w}}+m} \cdot (w-\dot{\xi}_c) \quad \text{and} \quad \frac{1/2\ Z_{w^2}}{-Z_{\dot{w}}+m} \cdot (w-\dot{\xi}_c) \cdot |w-\dot{\xi}_c| \tag{4.21}$$

b) roll

$$\frac{k_p}{-k_{\dot{p}}+I_{xx}} \cdot (p - \dot{\varphi}_{y_c}) \quad \text{and} \quad \frac{1/2\ k_{pa}}{-k_{\dot{p}}+I_{xx}} \cdot (p - \dot{\varphi}_{y_c})|p - \dot{\varphi}_{y_c}| \qquad (4.22)$$

where
$$\dot{\rho}_c = \frac{\delta \rho_c}{\delta t}$$
$$\dot{\varphi}_{y_c} = \frac{\delta \varphi_{y_c}}{\delta t}$$

When considering the non-linear coupled heave and roll mode, all the coupling terms as mentioned in Chapter 3 had to be added from which only the pure hydrostatic terms could be predicted by using, for example, the method presented in References[6, 7]. The same objections would apply with respect to the excitation.

In summary, the excitation due to the action of plane progressive waves can be calculated quite accurately using the Haskind relations, being the action from the diffraction potential calculated for each station by two dimensional techniques and the resultant force and moment obtained by integration of those two dimensional results along the length of the vehicle. Due to computational difficulties another method was used which gives results believed to be less accurate but offers the advantage of its simplicity. Another advantage not mentioned before is that this later method express

the exciting forces and moments in terms of instantaneous values of surface displacement and wave slope which are easy to obtain during the tank experiments setup to measure displacements and velocities that will be used in the identification procedure.

CHAPTER 5

EXPERIMENTAL PROCEDURE

A model of the "mariner" type hull form, whose characteristics are given in Table 5.1, was tested in regular waves measuring the displacements and velocities in heave, roll and coupled heave and roll, and also the surface elevation, with the purpose of identifying the hydrodynamic coefficients in those modes of motion.

TABLE 5.1 - MODEL CHARACTERISTICS

scale (model/full size)	1/96
length (L)	5.50 ft.
beam (B)	.79 ft.
draft (T)	.28 ft.
weight (W_o)	46.05 lb.
mass (m)	1.43 lb.sec²/ft.

The model was placed in the mid-length of the tank with its centerline perpendicular to the centerline of the tank such that, using the coordinate system as defined in Chapter 2, the direction of propagation of the waves generated at the wavemaker in the beginning of the tank, coincide with the negative $x_2 = y$ direction.

The model was placed at the mid-length of the tank

as a solution of compromise: not too close to the wavemaker in order to avoid the action of the diffracted waves from the model upon reflexion on the wavemaker, not too far from the wavemaker in order to avoid interference from the walls of the tank. Reflexion on the opposite end of the wavemaker was of much less concern due to the existence of a "beach" composed of stainless steel shavings with a reflexion coefficient of about .05, for normal wave lengths.

Tests were run at three different frequencies. First it was decided that those frequencies would be half, twice and the natural undamped frequency for both heave and roll, where the natural undamped frequency is defined as for a linear system

$$\omega_n = \sqrt{k/m} \qquad \text{where} \quad k = -Z_{zo} = -k_\phi$$

$$m = -Z_{\dot{w}} + m = -K_{\dot{p}} + I_{xx}$$

For roll the measured natural period (from free oscillations of the model in calm water) was $T_n = 1.52$ seconds or $f_n = .66$ cps. Therefore tests were run at

$f_1 = .33$ cps $\qquad \lambda = gT^2/2\pi \qquad = 47.36$ ft.

$f_2 = .66$ cps $\qquad\qquad\qquad\qquad = 11.84$ ft.

$f_3 = 1.32$ cps $\qquad\qquad\qquad\qquad = 2.96$ ft.

For heave the measured natural period was $T_n = .78$ seconds or $f_n = 1.28$ cps. For the tests to be run at twice the natural frequency $f = 2.48$ cps, it would be too close to the maximum at 2.50 cps permitted by the wave maker. Therefore, it was decided to run the tests in heave at one fourth, half and at the natural frequency, or

$$f_1 = .32 \text{ cps} \qquad \lambda = 49.92 \text{ ft.}$$
$$f_2 = .64 \text{ cps} \qquad = 12.48 \text{ ft.}$$
$$f_3 = 1.28 \text{ cps} \qquad = 3.12 \text{ ft.}$$

For the tests in coupled heave and roll, it was decided to run at the natural frequency in heave and at half and at the natural frequency in roll, therefore, for the coupled mode

$$f_1 = .33 \text{ cps} \qquad \lambda_1 = 47.36 \text{ ft.}$$
$$f_2 = .66 \text{ cps} \qquad \lambda_2 = 11.84 \text{ ft.}$$
$$f_3 = 1.28 \text{ cps} \qquad \lambda_3 = 3.12 \text{ ft.}$$

The quantities measured in each test were

a) heave

surface displacement $\zeta(t)$

heave displacement $z_o(t)$

heave acceleration $\ddot{z}_o(t)$

The reason for measuring acceleration instead of velocity was due to the impossibility of obtaining a translational velocity transducer, and to not being able to build a reliable electronic integrator with the material available

b) roll

surface displacement	$\zeta(t)$
roll displacement	$\phi(t)$
roll velocity	$p(t)$

Again wave slope should be measured in this mode instead of surface elevation but the probe used could only measure $\zeta(t)$ [1]

c) coupled heave and roll

surface displacement	$\zeta(t)$
heave displacement	$z_o(t)$
heave acceleration	$z_o(t)$
roll displacement	$\phi(t)$
roll velocity	$p(t)$

The same observations as made in a) and b) above apply.

[1] Description of probes with provision to measure simultaneously surface displacement and wave slope is available in Reference [24].

Surface displacement was measured with resistance type wave probe formed by two parallel stainless steel wires of 1/8" diameter. Since it was necessary to obtain the wave displacement at the vehicle, the ideal position for the wave probe would be along the centerline of the model. Since at this position the probe would be subject to interference between the model and the tank walls, it was decided to put the probe in integer number of wave lengths ahead of the model. Those distances were calculated based on the relation $k = \omega^2/g$ or $\lambda = gT^2/2\pi$ and this procedure introduced the first error on the measurements since for finite water depths the relation between wave number and frequency is [23]

$$\frac{\omega^2}{g} = k \tanh(kh) \qquad (5.1)$$

where h is the water depth.

For example, considering $h = 4$ ft.

$f = \omega/2\pi$	$(\lambda_\infty - \lambda_h)/\lambda_\infty$
.5 cps	.098
.65 cps	.025
7.85 cps	≈ 0

From the above it is seen that for the low frequencies f< .5 cps the error becomes appreciable.

Heave displacement was measured with a linear variable differential transformer (LVDT) connected to the heave rod assembly. The heave rod slides on a sleeve provided with ball bearings to reduce friction. Even though the friction was found to be quite high (comparing with experiments had with a heave rod assembly provided with air bearings) and therefore there will be some error involved in the identification of the heave damping parameters.

Heave acceleration was measured with an accelerometer placed in the model as close as possible to the axis of rotation in order to minimize interference of roll in the measurement of heave acceleration.

Roll displacement was measured by a rotational variable differential transformer attached to a roll bearing.

Roll velocity was measured by means of a rate gyro, borrowed from the Charles Draper Laboratory, which was placed in the model with its three axes of rotation parallel to the coordinate axes located on the model. This rate gyro, used in the Deep Submergence Systems Project, was able of measuring angular rates in roll, pitch and yaw simultaneously, up to rates of 40 degrees/second.

All the transducers mentioned above, with the exception of the rate gyro, are excited by an A.C. voltage of 5 volts at a frequency of 2400 cps provided by carrier preamplifiers Sanborn model 8805A which also received the electrical response from the transducers, generating a D.C. signal which may be fed to a chart or tape recorder or both. The amplifiers also provide means of amplifying or attenuating the electrical signal according to its level since both recorders work between ± 2.5 volts.

Since the excitation necessary for the rate gyro is 26 V A.C. (f = 400 cps), the Sanborn preamplifiers could not be used. Instead the following circuit was used (see Fig. 5.1), since a D.C. power supply was available up to three amps:

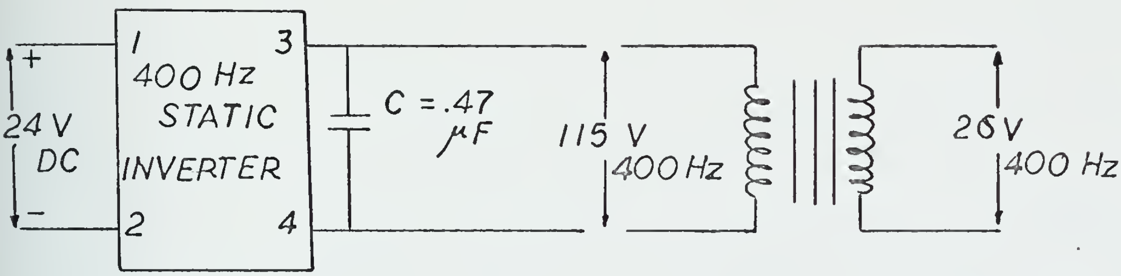

Figure 5.1

The 26 V 400 hz excitation was fed to the rate gyro. The output from the rate gyro (400 hz, phase

reversing) was then fed to a demodulator (PRD type 808) to obtain the D.C. signal proportional to the angular velocity. This signal was attenuated in a Sanborn D.C. amplifier before going into the chart and tape recorder (see Fig. 5.2).

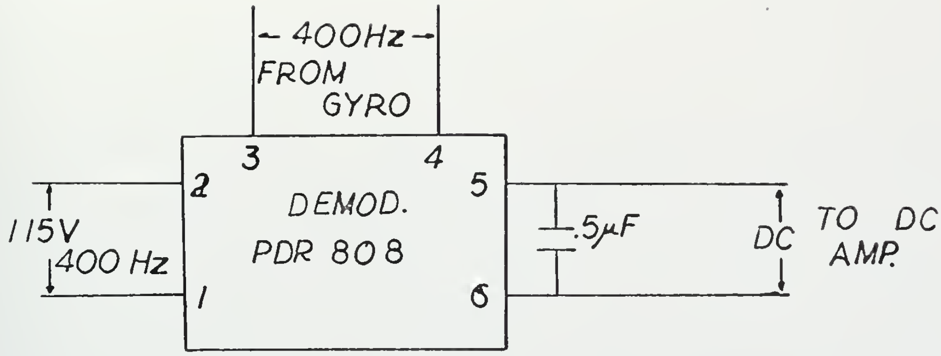

Figure 5.2

For convenience of future calculation, it was decided to put the center of gravity at the design waterline (all stability calculations were made for an assumed KG = .28 ft. - center of gravity at the waterline). This was performed by trial and error by creating a known moment (with the model in the upright position, a given weight is moved transversally a certain given distance) and measuring the resulting inclination which was compared

with the value calculated in the curves of stability. Weights were then moved in the vertical and the angle verified again until reaching the value specified in the curves.

Calibration of all instrumentation was made in the tank except for the rate gyro which was performed at the Draper Laboratory.

The LVDT was first zeroed by sliding the core into the casing until no output was shown on the chart recorder. This point was marked on the shaft of the core together with marks spaced 1" on each side. Then for a movement of the core of 1" and for a given attenuation of the pre-amplifiers the pen deflection on the chart recorder was noted (1" = 10 mm - 50 mv/mm attenuation factor). For the wave probe and rotational differential transformer the procedure was similar. For the rotational transformer using a protractor designed for that purpose. The calibration of the heave rod and roll bearing was checked later with the instrumentation aboard.

For the calibration of the accelerometer zero was checked with it in the horizontal. The accelerometer was then turned 90 degrees and its full scale value (1.5 g <> 48.3 ft./sec.^{2}) given full scale deflection of

the pen (25 mm). Then the zero suppression control on the preamplifier was adjusted to bring the output to zero.

For the rate gyro, the whole assembly (rate gyro, inverter, transformer and demodulator) was placed in a rotating table to which a known constant angular velocity was given. The output voltage was then measured with a high precision D.C. voltmeter. It was noted that the presence of demodulator introduced some hysteresis on the output as well as some non-linearity above an angular velocity of 20 degrees/second. On the average the output signal was of .136V/degree/second D.C. while for the rate gyro alone it was .147V/degree/second rms. Afterwards in the tank a constant angular velocity was simulated by means of a potentiometer, the output voltage read with an oscilloscope and the pen deflection on the chart recorder noted.

The results of the final calibration for all the instruments were as follows:

wave probe	1" = 10 mm (atten. factor = 10)
heave rod	1" = 10 mm (atten. factor = 50)
accelerometer	1.5 g = 25 mm (atten. factor = 50)
roll bearing	10° = 10 mm (atten. factor = 200)
rate gyro	20°/sec. = 10 mm (atten. factor = 200)

From the above some other mistakes can be noted. First, full scale (25 mm) corresponds to 2.5 volts. For a model of 3.375 inches of draft heave displacements of more than 1 inch (1/3 T) can hardly be expected. Therefore, the output signal on tape will vary between ± 1 volt at most. For the acceleration if it is assumed that, as for linear systems $\ddot{z}_a = \omega^2 z_a$ the amplitude of the acceleration would be for the highest frequency (f = 1.28 cps = 8.04 sec.$^{-1}$) z_a = 64.64 inch/sec.^{2} or 5.38 ft./sec:^{2} since full scale was 1.5 g = 48.3 ft./sec.^{2} the signal on tape would vary between ± .25 volts. For the lower frequencies the acceleration output was hardly perceptible on chart paper and not usable for the digitization, since output signal approached the noise level.

Similarly for roll displacement and velocity and for the wave probe, the signal level was quite low.

Before running the tests calibration signals were sent into the tape recorder in order to be able to scale the data to correct values of displacements and velocities once the digitization being performed. About 50 ft. of tape were recorded with those calibration signals at a recorder speed of 7½ inch/sec.

Then the nine tests as mentioned before were run

recording about 50 cycles for each run. In between runs enough time would elapse in order for the model and the water to calm down (zero initial conditions). On tape the interval between runs was about 20 ft.

The waves were generated by a wave maker constituted by a hinged paddle actuated by an electro-hydraulic power system, which was controlled by electrical sinosoidal signals generated by a low frequency wave oscillator, through a hydraulic servo valve.

The data was digitized from the magnetic tape into computer cards by means of the EAI 680 analog computer, EAI 693 data conversion system and the 1130 IBM digital computer of the Department of Mechanical Engineering. During digitization some more deficiencies were noted, besides the low level of the signals on tape (the EAI 680 works within a voltage level of ± 10.5 volts in normal operation; when digitized these signals correspond to a standard precision real value of ± 1.05). First the magnetic tape had, on the portions not recorded, a D.C. signal of about 5.8 volts whose origin couldn't be determined. A signal of this magnitude could magnetize the heads of the tape recorder when on "play", thus introducing errors on the values of the measurements.

Also the runs were too short to be possible to set up the values of the potentiometers conveniently. These potentiometers were used to take off any bias that existed in the measurements.

For purposes of digitization, these circuits were built in the EAI 680 analog computer (refer to Appendix 4 and Reference [25] for details in the circuitry):

1. - Control of Sampling Rate

A Wavetek 116 VCG oscillator is used to generate a square wave of frequency equal to the sampling rate. This signal is obtained in analog trunks 0 or 3. From the trunks the signal goes into the input of a comparator in the interface tray of the analog rows. The output, which is obtained in the same interface tray in the logic row is fed into a differentiator to obtain a spike. The output of the differentiator is in turn fed into the analog to digital converter (ADC) sample control terminals on the first strip, preferably on trunk "all". This signal may also be fed to a BDC counter in Column 01 to check the operation of the circuit.

2. - Signal Processing

From the tape recorder the signals are fed into analog trunks 14 to 20, located below the Wavetek

oscillator. From the trunks whose output is situated in the second strip the signal goes into a summer together with the output of a potentiometer (to take off any bias on the signal). The output of the summer is fed into a potentiometer and from the potentiometer into an amplifier (input marked with a 10). The purpose of the potentiometer is to attenuate, if necessary, the signal which is amplified ten times in the amplifier. The signal correspondent to the measured acceleration, instead of going into an amplifier, is fed into an integrator to convert it to velocity. The output of the amplifier or integrator is then fed into the ADC trunks (located in the second strip) to be digitized and at the same time into the display trunks for display into the CPS DU390T oscilloscope.

3. - Display Circuit

From terminal B for example, in the mode control timer (in column 02 of the logic row) a wire is connected to the "scope" trunks - DIS TRG terminal located in the first strip in order to obtain a display in any analog mode control. A pin is introduced in the "run" terminal located close to terminal B mentioned above.

Digitization was initiated in the IBM 1130 by a computer program built according to the hybrid user's guide published by the Department of Mechanical Engineering using existing routines. This computer program is presented in Appendix 4.

The output from the digitization was punched into computer cards.

The selected sampling rate was twenty samples/ cycle for fifty cycles for all runs. Such a frequent sample will allow a method of linear interpolation to be used in the identification of the parameters.

In summary the main points in the experimental procedure were outlined. The mistakes made, due mainly to inexperience in this kind of experiments, were also pointed out. If time permitted they should be repeated in order to obtain better data.

Digitization was explained in some extent in order to allow a somewhat easier handling of data in the future.

CHAPTER 6

IDENTIFICATION OF SHIP MOTION MODEL PARAMETERS

Most of the available techniques of parameter identification have been developed for linear dynamic systems [1]. Some of those techniques, assuming the linearized equations of motion to be a valid description of the behaviour of an ocean vehicle, have already been applied in the determination of ship motion parameters in heave and pitch namely, the frequency response [27] and the step response methods [28].

In the case of a non-linear model, two approaches are open, both yielding approximate results.

The first, making no assumptions about the characteristic of the uncertainty, assumes a mathematical model for the system and then comparing the output of both model and systems to the same input, searches for the structure of the model that minimizes a function of the error between the two outputs. An example of the foregoing is the model reference technique. For the system:

$$
\begin{aligned}
\underline{y}_s(t) &= \underline{f}_s(\underline{u}(t_o,t),t) \\
\underline{y}_m(t) &= \underline{f}_m(\underline{u}(t_o,t),t) \qquad (6.1) \\
V &= F(\underline{y}_s-\underline{y}_m)
\end{aligned}
$$

where $\underline{u}$ vector input to both system and model

$\underline{y}_s$ - vector output of the system

$\underline{y}_m$ - vector output of the model

$\underline{f}_s$ - vector structure of the system

$\underline{f}_m$ - vector structure of the model

V - scalar function to be minimized.

A simplified version of the above will be

$$\begin{aligned} \underline{y}_s(t) &= \underline{f}(\underline{u}(t_o,t),\ t,\ \underline{p}_s) \\ \underline{y}_m(t) &= \underline{f}(\underline{u}(t_o,t),\ t,\ \underline{p}_m) \end{aligned} \tag{6.2}$$

where the values of $\underline{p}_s$ are unknown and the values of $\underline{p}_m$ will be determined by minimizing V, which is often of the form

$$V(\underline{p}_m, t_o, t) = \frac{1}{t-t_o} \int_{t_o}^{t} [\underline{y}_s(\tau)-\underline{y}_m(\tau)][\underline{y}_s(\tau)-\underline{y}_m(\tau)]^T d\tau \tag{6.3}$$

and for optimum $\underline{p}_m$ (best approximation) $\frac{\delta V}{\delta \underline{p}_m} = 0$. The vector $\underline{p}$ thus found will only be optimum in the interval $t_o < \tau < t$ and may not apply outside that interval. Also it may not be true for non-linear systems that

$$\lim_{t \to \infty} \underline{p}_m = \underline{p}_s \tag{6.4}$$

as one might expect from linear systems. The optimum value of $\underline{p}_m$ in Equation (6.3) could be found by a maximum gradient seeking algorithm (first order gradient method) or by a second order gradient method [29]

The second approach is to use linear estimation techniques, such as a Kalman filter to the linearized form of a modified state representation of the system.

Considering a linear system driven by noise, Bryson and Ho [29, Chp. 11 and 12] present the following expressions for the estimate of the system:

1) Linear Multistage Systems

$\underline{x}_{i+1} = \underline{F}_i \underline{x}_i + \underline{G}_i \underline{w}_i$

$\underline{x}$ - state vector $\qquad E[\underline{x}_o] = \overline{\underline{x}}_o$

$$E[\underline{x}_o - \overline{\underline{x}}_o][\underline{x}_o - \overline{\underline{x}}_o]^T = M_o \qquad (6.5)$$

$\underline{w}$ - environment noise $\qquad E[\underline{w}] = \overline{\underline{w}}$

$$E[\underline{w}_i - \overline{\underline{w}}_i][\underline{w}_j - \underline{w}_j]^T = Q_i \delta_{ij}$$

$$E[\underline{w}_i - \overline{\underline{w}}_i][\underline{x}_o - \overline{\underline{x}}_o]^T = 0$$

$\underline{F}_i \; \underline{G}_i$ - transition matrices

and $\underline{z}_i = \underline{H}_i \; \underline{x}_i + \underline{v}_i$

$\underline{z}$ - measurement vector

$\underline{H}_i$ - measurement matrix $\qquad (6.6)$

$\underline{V}_i$ - measurement noise $\qquad E[\underline{v}_i] = 0$

$$E[\underline{v}_i \underline{v}_j^T] = R \delta_{ij}$$

$$E[\underline{x}_o - \overline{\underline{x}}_o] \underline{v}^T = 0$$

Assuming the sequences i of the different vectors to represent Gauss-Markov random sequences, they present the solution for the estimate of the state $\underline{x} \longrightarrow \hat{\underline{x}}$ as:

$$\hat{\underline{x}}_i = \overline{\underline{x}}_i + \underline{K}_i(\underline{z}_i - \underline{H}_i\overline{\underline{x}}_i)$$

$$\overline{\underline{x}}_{i+1} = \underline{F}_i \hat{\underline{x}}_i + G_i w_i$$

$$\underline{K}_i = \underline{P}_i H_i^T R_i^{-1} \tag{6.7}$$

$$\underline{P}_i = [\underline{M}_i^{-1} + \underline{H}_i^T \underline{R}_i^{-1} \underline{H}_i]^{-1} =$$

$$\underline{M}_i - \underline{M}_i\underline{H}_i^T(\underline{H}_i\underline{M}_i\underline{H}_i^T + \underline{R}_i)^{-1} \underline{H}_i\underline{M}_i$$

$$\underline{M}_{i+1} = \underline{F}_i\underline{P}_i\underline{F}_i^T + \underline{G}_i\underline{Q}_i\underline{G}_i^T$$

where $\underline{P}_i$ is the error covariance matrix of $\underline{x}_i$ after the measurement while $\underline{M}_i$ is the error covariance matrix before the measurement. Similarly $\hat{\underline{x}}_i$ is the estimate of the system after the measurement while $\overline{\underline{x}}_i$ is the estimate of the system before measurement.

2) Linear Continuous Systems with Continuous Measurements

The mathematical expression for this type of system is obtained by letting the time between stages go to zero. Therefore, for the system

$$\dot{\underline{x}} = \underline{Fx} + \underline{Gw}$$
$$\underline{z} = \underline{Hx} + \underline{v} \tag{6.8}$$

with
$$\underline{F} = \lim_{\Delta \to 0} \frac{F(t_i + \Delta, t_i) - \underline{I}}{\Delta} \qquad (6.8)$$
$$\underline{G} = \lim_{\Delta \to 0} \underline{G_i}$$

Under the assumption that $\underline{x}$ represents a Gauss-Markov randon process, the expression for the Kalman filter will be

$$\dot{\hat{\underline{x}}} = \underline{F\hat{x}} + \underline{G\bar{w}} + \underline{K}(\underline{z} - \underline{H\hat{x}})$$
$$\underline{K} = \underline{P}\,\underline{H}^T\,\underline{R}^{-1}$$
$$\dot{\underline{P}} = \underline{F}\,\underline{P} + \underline{P}\,\underline{F}^T + \underline{G}\,\underline{Q}\,\underline{G}^T - \underline{P}\,\underline{H}^T\,\underline{R}^{-1}\,\underline{H}\,\underline{P}$$
$$P(t_o) = P_o$$

If, for the system as specified in (1.11) to (1.13)

$$\dot{\underline{x}}' = \underline{f}'(\underline{x}', \underline{u}, p') + \underline{w}' \qquad (1.11)$$
$$\dot{\underline{p}} = 0 \qquad (1.12)$$
$$\underline{z} = \underline{H}'\,\underline{x}' + \underline{v} \qquad (1.13)$$

the above technique is to be used, it is necessary by first to form the extended state vector

$$\underline{x} = \begin{bmatrix} \underline{x}' \\ \underline{p} \end{bmatrix} \qquad (6.10)$$

The new state equations are as follows:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}) + \underline{w} \qquad E[x(t_o)] = \bar{\underline{x}}_o$$
$$\underline{z} = \underline{H}\,\underline{x} + v \qquad (6.11)$$

where in addition to 6.10

$$\underline{w} = \begin{bmatrix} \underline{w}' \\ \underline{0} \end{bmatrix} \qquad (6.12)$$

$$\underline{f} = \begin{bmatrix} \underline{f}' \\ \underline{0} \end{bmatrix}$$

$$\underline{H} = [\underline{H}' \quad \underline{o}] \tag{6.12}$$

Now the state equations are linearized in order to use Equation (6.9) or:

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u}) + \overline{\underline{w}} + \underline{K}(\underline{z}-\underline{H}\ \underline{x}); x(t_o) = \overline{\underline{x}}_o$$

$$\underline{K} = \underline{P}\ \underline{H}^T\underline{R}^{-1} \tag{6.13}$$

$$\dot{P} = \frac{\delta \underline{f}}{\delta \underline{x}}\ \underline{P} + \underline{P}\ \frac{\delta \underline{f}}{\delta x}^T + Q - \underline{P}\ H^TR^{-1}HP;\ \underline{P}(t_o) = \underline{P}_o$$

The next simplification will be, considering that in practice most measurements are discrete in time, to approximate the above differential equations by a set of difference equations and use the formulation for a linear multistage processes or, in other words, for the system below :

$$\underline{x}_{i+1} = \underline{x}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i) + \Delta \cdot \underline{w}_i \tag{6.14}$$

$$\underline{z}_i = \underline{H}\ \underline{x}_i + \underline{v}$$

The resulting filter will be, assuming for convenience that

$$E[\Delta \cdot \underline{w}_i] = \underline{0}$$

$$\hat{\underline{x}}_{i+1} = \overline{\underline{x}}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i) + \Delta \cdot \underline{K}_{i+1}(\underline{z}_i - \underline{H}\ \overline{\underline{x}}_i) \tag{6.15}$$

$$\underline{K}_i = \underline{P}_i \underline{H}^T (\underline{R} \cdot \Delta)^{-1}$$

$$\underline{P}_i = \underline{M}_i - \Delta\cdot\underline{M}_i\underline{H}^T(\underline{H}\ \underline{M}_i\ \underline{H}^T\Delta + \underline{R}\cdot\Delta)^{-1}\ \underline{H}\ \underline{M}_i$$

$$\underline{M}_{i+1} = \frac{\delta}{\delta\underline{x}_i}(\underline{x}_i+\Delta\cdot\underline{f}(\underline{x}_i,\underline{u}_i))\ \underline{P}_i\ \frac{\delta}{\delta\underline{x}_i}(\underline{x}_i+\Delta\cdot\underline{f}(\underline{x}_i,\underline{u}_i))^T + \underline{Q}\cdot\Delta \tag{6.15}$$

where $\underline{R}$ and $\underline{Q}$ are constant matrices. If the value of Δ is sufficiently small, this formulation will represent closely Equations (6.13) and will be much easier computationaly.

In the above formulations, the partial derivatives may be evaluated in a nominal path or if greater accuracy is desired they are evaluated at $\underline{x} = \hat{\underline{x}}$.

Bryson and Ho [29] mention several authors who have achieved success using Equations (6.13)and (6.15) for non-linear systems but at the same time make the warning that convergence may not be obtained if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system.

From the foregoing Equations (6.2) and (6.3), Equations (6.13) and Equations (6.15) it was decided to use Equations (6.15) due mainly to computational convenience. Due also to the poorness of the available data, as mentioned in Chapter 5, it was decided to abandon the identification in heave and coupled heave and roll

and concentrate exclusively on the identification of roll motion parameters whose data at a frequency of excitation of .66 cycles/second (roll natural undamped frequency) seemed reasonably good.

As mentioned in Chapter 4, Equation (4.17) the state equations for roll motion, ignoring for simplicity the dependency of the system on the derivative of the input are:

$$\dot{p} = \frac{K_p}{-K_{\dot{p}}+I_{xx}}\, p + \left[\frac{K_\phi}{-K_{\dot{p}}+I_{xx}} - \frac{W.BG}{-K_{\dot{p}}+I_{xx}}\right](\phi - f_y) + \frac{W.BG}{-K_{\dot{p}}+I_{xx}} + \left[\frac{1/2!K_{p|p|}}{-K_{\dot{p}}+I_{xx}}\right] p|p| + \left[\frac{1/3!K^{3}}{-K_{\dot{p}}+I_{xx}} + \frac{1/3!W.BG}{-K_{\dot{p}}+I_{xx}}\right](\phi - f_y)^3 + -\left[\frac{1/3!W.BG}{-K_{\dot{p}}+I_{xx}}\right]\phi^3 \qquad (4.17)$$

$$\dot{\phi} = p$$

Non-dimensionalizing the above equations with the frequency of excitation the following is obtained:

$$\left[\frac{\dot{p}}{\omega^2}\right] = \left[\frac{K_p}{-K_{\dot{p}}+I_{xx}} \cdot \frac{1}{\omega}\right]\left[\frac{p}{\omega}\right] + \left[\frac{K}{-K_{\dot{p}}+I_{xx}} \cdot \frac{1}{\omega^2} - \frac{W.BG}{-K_{\dot{p}}+I_{xx}}\,\frac{1}{\omega^2}\right][\phi - f_y] + \left[\frac{W.BG}{-K_{\dot{p}}+I_{xx}}\,\frac{1}{\omega^2}\right]\phi + \left[\frac{1/2!K_{p|p|}}{-K_{\dot{p}}+I_{xx}}\right]\frac{p}{} \cdot \left|\frac{p}{}\right| \qquad (6.16)$$

$$\pm\left[\frac{1/3!K^{3}}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2}+\left|\frac{1/3!W.BG}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2}\right](\phi-\phi_s)^3-\left[\frac{1/3!W.BG}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2}\right]\phi^3 \tag{6.16}$$

$$\left[\frac{\dot\phi}{\omega}\right]=\left[\frac{p}{\omega}\right]$$

Based on results of Appendices 1, 2 and 3, the following non-dimensional parameters will be, for $\omega = 4.145$ sec.$^{-1}$:

$$\frac{K_p}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega} = -.0042 = .01\ x(3)$$

$$\frac{K_p}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2} = -.989 = x(4)$$

$$\frac{W.BG}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2} = 2.744 = 10\ x(5) \tag{6.17}$$

$$\frac{1/2!K_{p\,p}}{-K_{\dot p}+I_{xx}} = -.126 = x(6)$$

$$\frac{1/3!K^{3}}{-K_{\dot p}+I_{xx}}\;\frac{1}{\omega^2} = 3.123 = 10\ x(7)$$

Also and to have the values of all the state variables between ±1, it was chosen that

$$\frac{\dot p}{\omega^2} = .1\ y(1)$$

$$\frac{p}{\omega} = .1\ x(1) \tag{6.18}$$

$$\frac{\dot\phi}{\omega} = .1\ y(2)$$

$$= .1x(2)$$
$$= .1u(1) \qquad (6.18)$$

The complete set will then have the form :

$$.1y(1) = .01x(3).1x(1) + [x(4)-10x(5)] [.1x(2)-.1u(1)] +$$
$$+ 10x(5).1x(2) +$$
$$+ x(6).1x(1).1|x(1)| - [10x(7)-.167\ 10x(5)]$$
$$[.1x(2)-.1u(1)]^3 - .167\ 10x(5)[.1x(2)]^3$$
$$.1y(2) = .1x(1)$$

or, rearranging

$$y(1) = .01\ x(3)\ x(1) + [x(4) - 10\ x(5)][x(2) - u(1)] +$$
$$+ 10\ x(5)\ x(2) + \qquad (6.19)$$
$$+ .1\ x(6)\ x(1)\ \mathrm{ABS}\ x(1) + [.1\ x(7) - .0167\ x(5)]$$
$$[x(2) - u(1)]^3 - .0167\ x(5)\ x(2)^3$$
$$y(2) = x(1)$$

Before using the data, this model was tested by simulating noisy measurements — only x(1) and x(2) were measurable in the simulation — and the control u(1), with a format similar to the one obtained in the data collected.

Also simplified forms of the state equations were tested. Their expression were :

<u>Non-linear equations of motion to Identify linear terms</u>

$$y(1) = .01\ x(3)\ x(1) + [x(4) - 10\ x(5)][x(2) - u(1)] +$$
$$+ 10\ x(5)\ x(2) + .0126\ x(1)\ \mathrm{ABS}(\ x(1)\ ) +$$
$$+ [.03123 - .0167\ x(5)][x(2) - u(1)]^3 + \qquad (6.20)$$
$$+ .0167\ x(5)\ x(2)^3$$

$$y(2) = x(1) \tag{6.20}$$

<u>Linear Equations of Motion</u>

$$y(1) = .01\, x(3)\, x(1) + [x(4) - 10\, x(5)]\, [x(2) - u(1)] + 10\, x(5)\, x(2) \tag{6.21}$$

$$y(2) = x(1)$$

Equations (6.19), (6.20) and (6.21) were tested varying the following parameters:

1. Initial State Estimate - The "true values" of the parameters (x(3) through x(7))were set in the simulation at the values given by Equation (6.17). The initial estimate was set either zero or Equation (6.17).

2. Initial State Estimate Error Covariance $\underline{P}_0$ - This is a diagonal matrix of equal elements which were set to 0, 2. and 5.

3. Order of Magnitude of the Elements of R Relative to the Elements of Q - Both these matrices are diagonal and of equal elements. The ratio of the elements of R to the elements of Q was set at 1, 2, 5 and 10.

4. Noise Level in Environmental Noise - $\underline{w}$ and in Measurement Noise - $\underline{v}$. Set at 0 and 1

Then the collected data was used in the identification procedure introducing small variations in the parameters mentioned in 1, 2 and 3 above around a reasonable set

obtained in the simulation to obtain the best possible convergence and also to test the effect of initial conditions - or in other words, the necessary accuracy in the theoretical prediction of the values of the parameters, to obtain convergence. Some of the results from the identification are presented in Appendix 5 while the discussion of the results is presented in Chapter 7.

Summarizing in this chapter some of the available techniques for parameter identification were discussed. Attention was focused in those that permit application to non-linear dynamic systems. From those, one was chosen that having perhaps less accuracy in the results, is easy to implement. The equations of motion for roll went then through the necessary modifications to be in agreement with the format of such formulation. The structure of the model and the formulation were tested by simulating the excitation and the measurements, and varying selected parameters to obtain a better insight of what to expect from the identification.

CHAPTER 7

DISCUSSION OF RESULTS

1. Simulation

The first equation to be tested was Equation 6.19. The noise level was set to zero, the ratio of elements of $\underline{R}$ to elements of $\underline{Q}$, from now on expressed formally as r/q by similarity with the scalar case, equal to $\underline{1}$, the initial state estimate error covariance matrix $\underline{\dot{P}}_0$ at [2] and the initial state estimate x_0 at zero (the "true value" was set in all simulations at the values expressed in equation (6.17)). The values of the estimate blew up and after 200 measurements the computer indicated overflow. This type of behaviour with no noise decided the use of Equations (6.20) or (6.21). These two models were both tested at no noise. Equations (6.20) were first tested with $\underline{x}(0) = $ Eq.(6.17), r/q = 1 and $\underline{P}_0 =$ [5] . The error on the estimate (XHAT - XX) where XHAT is the estimate and XX is the "true value" of the state, remained at zero during the whole experiment. However P(3,3) (variance of x(3) - linear damping coefficient) kept increasing with small oscillations. The other elements of $\underline{P}$ decreased quickly to less than .1 . The same behaviour was observed for Equations (6.21), showing that the influence of the

non-linear terms is small, at least for the no noise condition.

The last experiment in the no noise series was conducted with Equations (6.21) $\underline{P}_o$ = [2] , r/q = 1 and $(\underline{XHAT})_o$ = [0].

The error ($\underline{XHAT - XX}$) for all variables converged rapidly to zero - in less than 100 measurements with the exception of (XHAT(3) - XX(3)) whose value remained constant throughout. Again the term P(3,3) steadily increased; while all the other variances came to values, after 500 measurements, less than .060 (initial value was 2.0) the final value of P(3,3) was 2.202. The foregoing suggests that when no noise is present the system converges independently of initial conditions, with the exception of Equations (6.19) and with the exception of the linear damping coefficient x(3) whose identification seems very difficult to obtain.

The rest of the experiments were performed with noise level equal to 1 which is believed to be quite a high level due to the scattering observed in the measurement vector $\underline{z}$.

The first simulations were taken with r/q = 1 $\underline{XHAT}_o$ = [0] and $\underline{P}_o$ = [3] and = [5]. Both simulations blew up at about the 140th measurement. At that time P(3,3),

P(4,4) and P(5,5) were quite large, but different from one case to the other. The main point to be observed here is the conjugate effect of noise and not accurate initial estimate.

The next experiment put together Equations (6.20) and (6.21). For both systems r/q = 1, $\underline{P}_o$ = [0] and $(\underline{XHAT})_o$ = $(\underline{XX})_o$. Since initially the error covariance matrix was set to zero, it increased and then oscillated about values equivalent to the ones found in the no noise condition (less than .050), with the exception of P(3,3) which steadily increased to a final value at the 500th measurement of .249 in both cases, with tendency to increase. The errors on the variables oscillates around zero with the exception of (XHAT(3) - XX(3)) which remained constant at about .035. Again it seems that the identification of x(3) will be dificult to obtain.

Next an experiment was performed with Eq. (6.20) with r/q = 1, $(\underline{XHAT})_o$ = $(\underline{XX})_o$ but $\underline{P}_o$ = [5] which diverged again confirming the influence of the initial conditions when combined with noise.

The next set of experiments were performed with the noise level at 1, as before, but r/q = 2, 5 and 10.

Comparing the results from Equations (6.21) for

r/q 1 and r/q 2, $\underline{(XHAT)}_o = [0]$ and $\underline{P}_o = [2]$, in the later case the estimate does not blow up although the error on the estimate and the variance both increase for x(3) and x(4), while for x(5) the error on the estimate and the variance decreases, which demonstrates some improvement. The cases $r/q = 5$ $r/q = 10$ represent no advantage over $r/q = 2$, presenting the same trends.

Similar simulation was performed with Equations (6.20). Abain an improvement is noted when r/q goes up to two but no further improvement is obtained when $r/q = 5$ or 10. On the contrary, the decreasing in the error seems to take place more slowly in the later case. In the case $r/q = 2$ the advantage of considering the non-linear terms when identifying the linear parameters is noted because P(3,3), which increased in the case of Equations (6.21), is seen to oscilate between approximately 2.02 and 2.16, and P(4,4), after increasing for the first couple of measurement, markedly decreases. The foregoing results were then compared with the case of $\underline{P}_o = [5]$, with clear advantage for the former.

The foregoing seems to indicate that the values $r/q = 2$ and $\underline{P}_o = [2]$ are the best suited among all other combinations tested. At this stage small variations could be tried to further improve the convergence. But since the caracteristics of the data are not expected to follow very closely the caracteristics of

the simulation, it seems a better idea to try to identify the linear parameters with the data obtained from the experiments and then vary the values of $\underline{R}$, $\underline{Q}$ and $\underline{P}_o$ in order to obtain a better convergence. But before that, one last simulation was tried for the same values of $\underline{Q}$, $\underline{R}$ and $\underline{P}_o$, but for values of $(\underline{XHAT})_o$ closer to the true values : $(\underline{XHAT})_o = [0, 0, -.2, -.5, .15]$ (about 1/2 of the true values). The number of measurements was also increased to 1000. It was noted that :

i) P(5,5) decreased rapidly to a value of .4 (after 10 measurements) and then decreased oscillating until it reached a value of .057. The error on the estimate, after 1000 measurements, was oscillating between $\pm .1$.

ii) P(4,4) decreased less rapidly to a value of .566 (after 30 measurements) and then jumped to a value of .06 where it kept oscillating for about 350 measurements decreasing finally to a value of .025 where it stayed oscillating until the end. The error on the estimate oscillated between ± 1 for 400 measurements and then decreased to zero oscillating about that point.

iii) P(3,3) increased to a value of 2.22 after 400 measurements, oscillated for a while and finally decreased to reach at the end a value of 1.29, with tendency for further decrease. The error on the estimate

increased to a value of 3.3 after 420 measurements, oscillated about that point for a while and finally decreased, oscillating to a value of 1 at the end.

Note that the turnover point seems to be the 400th measurement and that a set of initial conditions on the estimate closer to the true value really improves convergence. This fact confirms the believe that, contrary to the linear case, the initial conditions on both the estimate and its covariance play an important in the success or failure of the identification.

2. Identification Using Experimental Data

Following the conclusions of last section a run was made with $\underset{\sim}{Q} = [.1]$, $\underset{\sim}{R} = [.2]$ and $\underset{\sim}{P}_o = [2.]$. The set of initial conditions was as prescribed by Equations (6.17). Simultaneously, runs with the values of $\underset{\sim}{Q}$, $\underset{\sim}{R}$ and $\underset{\sim}{P}_o$ as indicated below were also made. The set of initial conditions, however, was not changed at this stage. It seemed preferible, since the above set is believed to be, among the available predictions, the closest to the true value, to find first the best set of $\underset{\sim}{Q}$, $\underset{\sim}{R}$, and $\underset{\sim}{P}_o$ changing $(\underset{\sim}{XHAT})_o$ afterwards.

Values of $\underset{\sim}{Q}$, $\underset{\sim}{R}$ and $\underset{\sim}{P}_o$:

$$\underset{\sim}{Q} = [.1]$$

$$\underset{\sim}{R} = [.1] \; , \; [.2] \; , \; [.3]$$

$$\underset{\sim}{P}_o = [1.] \; , \; [2.] \; , \; [5.]$$

in the following combinations of $\underset{\sim}{R}$ and $\underset{\sim}{P}_o$:

([.05] , [2.])

([.1] , [2.]) , ([.1] , [5.])

([.2] , [1.]) , ([.2] , [2.]) , ([.2] , [5.])

([.3] , [1.])

For the group ([.2] , [2.]) the results obtained with the first measurements were more or less as expected. P(5,5) decreased rapidly, P(4,4) decreased less rapidly and P(3,3) increased steadily. Then P(4,4) came close to zero and became negative for a while, then crossed zero again and went up to 2.48, decreased to .270 and increased again to the final value of .421. Also at the end P(5,5) = .059 and P(3,3) = 2.473 . On the other runs a similar behaviour was noted. For $\underline{R} = [.2]$ and $\underline{P}_0 = [1.]$, P(4,4) crossed zero and kept decreasing up to a final value of -1.589. P(5,5) decreased to reach the value of .064 (at the same point at which P(4,4) became negative) and then increased to reach a value of .123 at the end. P(3,3) increased steadily to reach 1.462 at the end. For the case $\underline{R} = [.2]$, $\underline{P}_0 = [5.]$ it was even worse since at the end P(3,3) = -43,225 and XHAT(3) = 252,70... For $\underline{R} = |.1|$, $\underline{P}_0 = |5.|$, P(3,3) increased steadily, P(4,4) decreased to a value of .173 and then increased again to reach at the end .425 with XHAT = -14.513 (?). P(5,5) decreased to a value of .028 and kept oscilla -

ting about that value, but with XHAT(5) always increasing. For $\underline{R}=[.1]$, $\underline{P}_o=[2.]$ the behaviour was similar but not so extreme due to the smaller value of $\underline{P}_o$.

For $\underline{R}=[.05]$, $\underline{P}_o=[2.]$, P(3,3) always increased while P(4,4) and P(5,5) decreased first to low values (.60 and .20) and increased afterwards, never becoming negative. For the last run of this group ($\underline{R}=[.3]$, $\underline{P}_o=[1.]$), the type of behaviour was similar to ($\underline{R}=[.2]$, $\underline{P}_o=[1.]$) but much more extreme —— the last recorded value of P(4,4) was -226.1 at the 350th measurement because some cycles later the computer stopped by overflow.

The fact that P(4,4) became negative posed the question of the correctness of the computerized scheme. However the program after thourough checking have been working successfully with linear systems. Besides no such strange behaviour was found during the simulation.

Therefore the program was checked again in detail but nothing could be found able to explain the fore-[illegible] going.

It was sugested then that as P(4,4) approaches zero, the computer could introduce some errors thus making it negative. The process would be selfsustaining afterwards. Assuming that to be the cause, the solution proposed was, whenever P(I,I) became negative, to substitute it by its positive counterpart. Accordingly

the following statements were introduced in the Kalman filter (KLMN subroutine) :

```
          DO 21 I   1, NX
          IF( P(I,I).LT.0.0) P(I,I)= -P(I,I)
       21 CONTINUE
```

The effects of this procedure were then verified on the following combinations of $\underline{R}$ and $\underline{P}_o$:

([.2] , [1.])

([.2] , [5.])

([.3] , [1.])

For the second case, this had as a consequence to hold XHAT(4) within bounds longer than before, only to diverge more violently towards the end (XHAT(4) ≐ 735.). The other two parameters had also large values. For the first case, the influence of the above procedure is more evident, since originally XHAT(4) decreased, crossed zero at approximately the 100th measurement (point where P(4,4) became negative) and went on decreasing. With the presence of the above statements, it behaved in the same way for the first 100 measurements, then stayed at a value of .8 until the 300th measurement, starting a slow decrease through zero to reach the value of -2.605 at the end . Acordingly the variance P(4,4), after touching zero increased to a final value of .242 . The same type of behaviour is also noted in

both XHAT(3) — diverging with negative values before, with P(3,3) increasing and being constant now with P(3,3) increasing also, and XHAT(5) — going away from zero with positive values before and going away from zero but with negative values and at a much slower rate, with the introduction of the above statements. P(4,4) seemed to be bouncing between zero and some finite positive values, but since the values of P were only recorded every 50 measurements, no valid conclusion can be taken about that fact. P(3,3) always increased and P(5,5) remained at approximately .057 .

It was decided then to modify the structure of the state equations to check if another form of the equations would converge to a solution. In Equation (6.16), for example the two groups forming the coefficient of $x(2) - u(1)$ could be considered as one parameter, being the new forms of $x(4)$ and $x(5)$:

$$10\ x(4) = \frac{K - W.BG}{-K_{\dot{p}} + I_{xx}} \cdot \frac{1}{\omega^2}$$

$$10\ x(5) = \frac{W.BG}{-K_{\dot{p}} + I_{xx}} \cdot \frac{1}{\omega^2} \qquad (7.1)$$

And the new form of the state equations would then be :

$$\begin{aligned} y(1) = {} & .01\ x(3)\ x(1) + x(4)\left[x(2) - u(1)\right] \\ & + 10\ x(5)\ x(2) + .0126\ x(1)\ \mathrm{ABS}(\ x(1)\ ) \\ & + \left[.0312 + .0167\ x(5)\right]\left[x(2) - u(1)\right]^3 \\ & - .0167\ x(5)\ x(2)^3 \end{aligned} \qquad (7.2)$$

$y(2) = x(1)$

For comparison only the following valued of $\underline{R}$ and $\underline{P}_o$ were considered :

i) $\underline{R} = [.2]$ and $\underline{P}_o = [5.]$

ii) $\underline{R} = [.3]$ and $\underline{P}_o = [1.]$

Also and to check the behaviour of the system at high values of $\underline{Q}$ and $\underline{R}$, the following run was made :

iii) $\underline{Q} = [1.]$, $\underline{R} = [2.]$, $\underline{P}_o = [1.]$

The main caracteristics of these runs was that the parameters did not oscillate, or oscillated with a very long period. Only in the case $\underline{R} = [.2]$, $\underline{P}_o = [5]$, XHAT(3) went over 1. at the 310th measurement, approximately. XHAT(4) and XHAT(5) which started at -.373 and +.274 respectively, crossed each other and zero at the 520th measurement reaching at the end the values +.359 and -.422 and showing in between a behaviour completely linear. P(3,3) increased to a value of 5.034 at the 100th measurement and then decreased as steadily as it had increased to a final value of 3.412. Similarly, P(4,4) increased to a value of 5.003 in the first 15 measurements and then decreased to a final value of 4.838. P(5,5) from the start decreased oscilla- ting until it reached a final value of .384 . This looked like being the object of the search apart from the fact that the values the parameters reached at the

end were not to be expected on physical grounds, specially XHAT(5) (this would mean that the center of bouyancy had to be above the center of gravity which is not true from the calculated curves). A possibility to consider is that the variables go through a cycle of very long period and therefore the convergence will take place later in the process. For the case $\underline{R}$ $[.3]$, $\underline{P}_o$ $[1.]$, only the value of P(5,5) decreased and the parameters changed their value very slowly and therefore nothing could be said about such a process. It could be an oscillatory motion of a period even longer than before or it could be a steady, but very slowly divergent process. In the last case considered, the parameters did not change but the diagonal elements of the covariance matrix increased and quite rapidly to reach values of $[1.91 , 2.239 , 48.289 , 48.195 , 32.620]$. Note that that the variance of XHAT(1) and XHAT(2) was, for all the cases considered, simulation included, of the order of .025 and .012 after the first 20 to 30 measurements. Therefore this last case is of no consequence.

In summary in this chapter the results from the identification were presented, both for the simulation carried out and for use of the data. It was pointed out of the difficulty of the system to converge. A set of values of the different covariance matrices was selected

from the simulation studies as being those that offered more possibilities when using real data. However the results didn't meet the expectations, being necessary to proceed in looking a set that would make the system converge . In stead it was found that some of the diagonal terms of P would become negative. First it was thought to be resultant from a mistake in the compute-rised scheme and later to errors introduced by the computer itself and corrected accordingly. Even then no solution could be found after 1000 measurements . Then another form of the state equations was obtained by combining two of the parameters. This later system of equations presented some interesting caracteristics, but nothing could be said in definite.

CHAPTER 8

CONCLUSIONS AND RECOMENDATIONS

From Chapter 7 it was quite clear that convergence of the estimate of the system to its true value was, in general, not possible to obtain. Reasons that could account for this failure in convergence were then pointed out as follows :

1 - Possible mistakes in the computerized scheme.

2 - Not accurate prediction of the initial conditions of the system.

3 - Data that measured the dynamical behaviour was not accurate enough.

4 - The input to the model didn't really have the same caracteristics as the input to the system in the experiments.

To those reasons above another may be pointed out that could be partly responsible for the non-convergence of the system, namely :

5 - Incompatibility between the system structure and the particular technique that was chosen .

With respect to the first reason, the fact , mentioned already that this same procedure was used before in the identification of linear static systems successfully, although not a suficient reason to

completely exclude this possibility, at least decreases the probability of such an event. Therefore it is recommended that if future use is intended to such a formulation, the program be rechecked for mistakes.

The not accurate prediction of the initial conditions can be a reason of some standing if the methods used for prediction fall under the category of empirical or semiempirical formulations from which the final result have not the reliability required. For the calculation of the motion parameters for roll, for example, the three categories in which Chapter 3 was divided also provide a good basis for classification of accuracy, starting with the hydrostatic coefficients at the top and the non-linear coefficients at the bottom of the scale. It is believed that for the case of excitation due to regular waves a formulation as presented by Ref. 18 and 19 for linear added mass, added moment of inertia and damping coefficient is completely adequate in the sense that the system will converge if the values of those parameters constitute the only approximation involved. This cinclusion is reached with information on the simulations carried out in Chapter 7.

When trying to digitize the data from the towing tank experiments, it was believed that this item

played a major role in the success or failure of the identification. This is not so anymore, when one compares the data from the tank with the measurements generated in the simulation, where the " level " of noise is much higher than in the former case. This is not the same as saying that it is not necessary to care about the experiments. If nothing else a careful planning will save a lot of time and labor, waisted otherwise trying to correct past mistakes.

The accurate determination of the input to the system as a function of time seems to play an important role in the success of the identification. Intuitively if the behaviour of two systems is to be compared, it is necessary that the input be the same. Unfortunately this was not the case. Both methods presented in Chapters 3 and 4 (using the Haskind Relations or assuming the major part of the excitation to be due to changes in bouyancy due to the passage of the wave), will not give very accurate results, the first because it does not present the instaneous values of the input, but an expression that assumes before hand an harmonic time dependency, the second because, although it expresses the input as the instantaneous function of a quantity being measured in the experiment, exprimes a very rough approximation of such dependence. However

the second was used in the identification procedure and it is still believed that, por this particular case the second yelds better results.

The incompatibility between the system structure and the method of identification exists because some of the assumptions set forth in the derivation of this particular method of identification were not met by the system. One of such assumptions was that the noise in the system, both environmental and measurement noise were uncorrelated with each other and with the state of the system (Equations 6.5 and 6.6). This last point was not met by the vehicle structure since, for example, finite water depth effects, interference between the vehicle and the walls, etc., would in one way or another be correlated with motions of the vehicle. This fact as a reason for failure to achieve convergence gained importance when comparing the results of the simulation with the results of the identification, where the former, being bad, were a bit more encouraging than the last.

With the above in mind, it seems that in order to improve the application of the identification techniques to problems of the type discussed herein, it will be necessary :

1 - Generate a vehicle environment that can be

better approximated as an uncorrelated random process. This sugests the possibility of using random seas to generate the data for the identification.

2 - Use another method of system identification, such as the model refference technique briefly described in Chapter 6, which do not make such restrictive assumptions.

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APPENDIX 1

HYDROSTATIC COEFFICIENTS

1 - DISCRIPTION OF THE COMPUTER PROGRAMS TO OBTAIN A POLYNOMIAL SURFACE REPRESENTATION

2 - VALUES OF THE HYDROSTATIC COEFFICIENTS FOR A MODEL OF THE " MARINER " HULL FORM

```
C      POLYNOMIAL SURFACE REPRESENTATION USING LEGENDRE POLYNOMIALS
C      IMPLEMENTED BY G. PARISSIS BASED ON KERWIN'S METHOD
C      DISCRIPTON AND FORMAT OF INPUT PARAMETERS
C      CARD  1        FORMAT(3I20)
C              IT  -  NUMBER OF DATA POINTS PER CURVE   (  MAX = 60  )
C              JT  -  TERMS IN THE POLYNOMIAL IN  Z     (  MAX = 20  )
C              MT  -  NUMBER OF CURVES TO BE FITTED     (  MAX = 25  )
C      CARD  2        FORMAT(2F20.5)
C              X1  -  LOCATION OF FIRST DATA POINT
C                     -.25L.E.X1L.E.0.0    IN THE NONDIMENSIONAL INTERVAL ( 0,1 )
C                     X1 = 1.0  IN THE NONDIMENSIONAL INTERVAL ( -1,1 )
C              X2  -  LOCATION OF LAST DATA POINT
C                     USUALLY X2 = 1.0
C      CARD  3        FORMAT(3I20)
C               K(N)  -  LOCATION OF SPECIAL TERMS IN  Z  -  N  =  1,2,3
C                     THE SPECIAL TERMS SHOULD ALWAYS BE THE LAST TERMS IN THE  Z
C                     POLYNOMIAL. THUS, IF JT = 10 AND THERE ARE THREE SPECIAL
C                     TERMS:    K(1) = 8  ,  K(2) = 9  ,  K(3) = 10 , IF JT = 10
C                     AND THERE ARE TWO SPECIAL TERMS : K(1) = 9  ,  K(2) = 10  AND
C                     THERE IS NO  K(3). IF THERE ARE NO SPECIAL TERMS THIS CARD
C                     REMAINS BLANK
C      CARD  4        FORMAT(3I20)
C               L(N)  -  EXPONENTS OF SPECIAL TERMS IN  Z  -  N  =  1,2,3
C                     MAX. NUMBER = JT . OTHERWISE NO RESTRICTION . WHEN THERE ARE
C                     LESS THAN THREE SPECIAL TERMS, WHAT WAS SAID IN CARD  3
C                     APPLIES
C      CARD  5  AND FOLLOWING CARDS      FORMAT(11F6.4)
C              DATA(I,M)     I=1, IT   M=1, MT         -  DATA POINTS
C                     INDICATE THE VALUES OF THE SURFACE TO BE FITTED STARTING AT
C                     LOCATION X1 AND ENDING AT LOCATUON X2 ( FIRST IN THE  Z  DI-
C                     RECTION AND THEN IN THE  FI  DIRECTION )
C              JT  -  NUMBER OF TERMS IN THE POLYNOMIAL IN X  (  MAX. = 20  )
C      CARD FOLLOWING THE LAST DATA CARD ( CARD  R )   FORMAT(I20)
C      CARD  R+1      FORMAT (2F20.5)
C              X1  -  LOCATION OF FIRST DATA POINT    X1  =  1.0
C              X2  -  LOCAION OF LAST DATA POINT  -  X2  =  -1.0
```

```
C      CARD  R+2     FORMAT(3I20)
C                K(N)  -  LOCATICN OF SPECIAL TERMS IN  FI  -  N  =  1,2,3
C                    SAME COMENTS AS FOR CARD  3
C      CARD  R+3     FORMAT(3I20)
C               L(N)  -  EXPONENTS OF SPECIAL TERMS IN  FI -  N  =  1,2,3
C                    SAME COMENTS AS FOR CARD  4
      DIMENSION K(3),L(3),DATA(28,28),A(28,20),H(28,28),D(20,20),C(20,28
     1),UNIT(20,20),R(28),SPACE(20),B(20,20),G(20,25),Q(20,25),E(28,20),
     2POLYN(25,25),T(20,20)
  200 FORMAT(73H  *NO OF DATA PER CURVE* *TERMS IN POLYNOMIAL OF Z* *NO
     1OF CURVES TO FIT*)
  201 FORMAT(3I20)
  202 FORMAT(63H  *LOCATION OF FIRST DATA POINT* * LOCATION OF LAST DATA
     1 POINT*)
  203 FORMAT(2F20.5)
  204 FORMAT(29H  *LOCATION OF SPECIAL TERMS*)
  205 FORMAT(30H  *EXPONENTS OF SPECIAL TERMS*)
  206 FORMAT(2H  ///8H  *DATA*)
  207 FORMAT(11F6.4)
  208 FORMAT(2H  ///68H  *EVALLATION OF LEGENDRE POLYNOMIALS IN Z AT THE
     1 INPUT DATA POINTS*)
  209 FORMAT(2H  ///59H  *EVALUATION CF COEFFICIENTS OF LEGENDRE POLYNOM
     1IALS IN Z*)
  210 FORMAT(4X,10E12.6)
  211 FORMAT(2H  ///42H  *RMS ERROR FOR EACH OF THE ABOVE CURVES*)
  212 FORMAT(10X,E12.6)
  213 FORMAT(2H  ///72H  *COEFFICIENTS OF POWERS OF Z STARTING WITH ZERO
     1ETH POWER AND GOING UP*)
  214 FORMAT(2H  ///34H  *TERMS IN THE POLYNOMIAL OF FI*      )
  215 FORMAT(2H  ///45H  *COEFFICIENTS OF LEGENDRE POLYNOMIALS IN Z*)
  216 FORMAT(2H  ///108H  *MATRIX OF COEFFICIENTS OF Z AND FI TERMS   RE
     1AD INCREASING POWERS OF Z HORIZONTALLY AND OF FI VERTICALLY*)
  217 FORMAT(I20)
  218 FORMAT(11F10.4)
  219 FORMAT(2H  ///24H  *TEST PRINT OF Q(J,M)*)
  222 FORMAT(2H  ///73H  *COEFFICIENTS OF LEGENDRE POLYNOMIALS A(M,N). N
```

```
     1 INCREASES HORIZONTALLY*)
       WRITE(6,200)
   300 READ(5,201)IT,JT,MT
       IF(IT)550,550,301
   301 WRITE(6,201)IT,JT,MT
       WRITE(6,202)
       READ(5,203)X1,X2
       WRITE(6,203)X1,X2
       WRITE(6,204)
       READ(5,201)(K(N),N=1,3)
       WRITE(6,201)(K(N),N=1,3)
       WRITE(6,205)
       READ(5,201)(L(N),N=1,3)
       WRITE(6,201)(L(N),N=1,3)
       IF(ABS(X2-X1)-1.5)500,500,501
   500 INTVAL=1
       GO TO 504
   501 INTVAL=2
   504 WRITE(6,206)
       READ(5,207)((DATA(I,M),I=1,IT),M=1,MT)
       WRITE(6,218)((DATA(I,M),I=1,IT),M=1,MT)
   516 CALL HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
       WRITE(6,208)
       WRITE(6,218)((H(I,M),I=1,IT),M=1,MT)
       WRITE(6,211)
       WRITE(6,212)(R(M),M=1,MT)
       WRITE(6,215)
       DO 610 M=1,MT
   610 WRITE(6,210)(C(J,M),J=1,JT)
       IF(K(1))510,510,511
   510 JJT=JT
       GO TO 512
   511 JJT=K(1)-1
   512 CALL LEGTR(JJT,INTVAL,B)
       DO 507 KK=1,JJT
       DO 507 MM=1,MT
```

```
      TERM=0.0
      DO 506 LL=1,JJT
  506 TERM=TERM+C(LL,MM)*B(LL,KK)
  507 G(KK,MM)=TERM
      K1=K(1)
      K2=K(2)
      K3=K(3)
      INT1=INTVAL
      IF(K(1))522,522,514
  514 DO 508 MM=1,MT
      G(K1,MM)=C(K1,MM)
      G(K2,MM)=C(K2,MM)
  508 G(K3,MM)=C(K3,MM)
  522 WRITE(6,213)
      DO 611 MM=1,MT
  611 WRITE(6,210)(G(KK,MM),KK=1,JT)
      MT1=MT
      JT1=JT
      IT=MT
      MT=JT
      WRITE(6,214)
      READ(5,217)JT
      IF(GT)550,550,525
  525 WRITE(6,217)JT
      WRITE(6,202)
      READ(5,203)X1,X2
      WRITE(6,203)X1,X2
      WRITE(6,204)
      READ(5,201)(K(N),N=1,3)
      WRITE(6,201)(K(N),N=1,3)
      WRITE(6,205)
      READ(5,201)(L(N),N=1,3)
      WRITE(6,201)(L(N),N=1,3)
      IF(ABS(X2-X1)-1.5)540,540,541
  540 INTVAL=1
      GO TO 542
```

```
541 INTVAL=2
542 DO 505 M=1,MT
    DO 505 I=1,IT
505 DATA(I,M)=C(M,I)
    CALL HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
    WRITE(6,209)
    DO 612 I=1,IT
612 WRITE(6,210)(H(I,M),M=1,MT)
    WRITE(6,211)
    WRITE(6,212)(R(M),M=1,MT)
    WRITE(6,222)
    DO 613 M=1,MT
613 WRITE(6,210)(C(J,M),J=1,JT)
    IF(K(1))531,531,532
531 JJT=JT
    GO TO 533
532 JJT=K(1)-1
533 CALL LEGTR(JJT,INTVAL,T)
    DO 535 KK=1,JJT
    DO 535 M=1,MT
    TERM=0.0
    DO 534 LL=1,JJT
534 TERM=TERM+C(LL,M)*T(LL,KK)
535 Q(KK,M)=TERM
    IF(K(1))536,536,537
537 KK1=K(1)
    KK2=K(2)
    KK3=K(3)
    DO 538 M=1,MT
    Q(KK1,M)=C(KK1,M)
    Q(KK2,M)=C(KK2,M)
538 Q(KK3,M)=C(KK3,M)
536 WRITE(6,219)
    WRITE(6,210)((Q(J,M),J=1,JT),M=1,MT)
    IF(K1)570,570,571
570 MMT=MT
```

```
      GO TO 572
  571 MMT=K1-1
  572 DO 574 MK=1,MMT
      DO 574 JJ=1,JT
      TERM=0.0
      DO 573 ML=1,MT
  573 TERM=TERM+Q(JJ,ML)*B(ML,MK)
  574 POLYN(JJ,MK)=TERM
      IF(K1)576,576,577
  577 DO 578 J=1,JT
      POLYN(J,K1)=Q(J,K1)
      POLYN(J,K2)=Q(J,K2)
  578 POLYN(J,K3)=Q(J,K3)
  576 WRITE(6,216)
      DO 579 JX=1,JT
      WRITE(6,210)(POLYN(JX,MZ),MZ=1,MT)
  579 CONTINUE
      GO TO 300
  550 CALL EXIT
      END
```

```
C     *** HUPOL2 ***             20 SEPTEMBER 1966
      SUBROUTINE HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
      DIMENSION  K(3),L(3),DATA(28,28),A(28,20),H(28,28),D(20,20),
     1  C(20,28),UNIT(20,20),E(28,20),R(28),SPACE(20)
C     THIS PROGRAM COMPUTES MODIFIED LEGENDRE POLYNOMIAL APPROXIMATIONS
C     H(X) TO UP TO 28 SETS OF INPUT DATA DATA(X). EACH CURVE MAY HAVE
C     UP TO 28 POINTS AND THE MAXIMUM NUMBER OF TERMS IS 20. ANY 3
C     OF THESE TERMS MAY HAVE ARBITRARY EXPONENTS SPECIFIED.
C     ******************************** ARGUMENTS ******************************
C      PROVIDED BY CALLING PROGRAM         COMPUTED BY HUPOL2
C     IT=NUMBER OF DATA POINTS/CURVE    A=COORDINATE FUNCTIONS
C     JT=TERMS IN POLYNOMIAL            H=POLYNOMIAL EVALUATED
C     MT=NUMBER OF CURVES TO FIT        D=NORMAL EQUATIONS AND INVERSE
C     K=LOCATION OF SPECIAL TERMS       C=COEFFS OF LEGENDRE POLYNOMIALS
C     L=EXPONENTS OF SPECIAL TERMS      UNIT=UNIT MATRIX TO GET INVERSE
C     X1=X VALUE OF FIRST DATA POINT    E=RIGHT HAND SIDE OF NORMAL EQNS
C     X2=X VALUE OF LAST DATA POINT     R=RMS ERROR FOR EACH CURVE
C     INTVAL=1 IF X RANGE NEAR (0,1)
C            2 IF X RANGE NEAR (-1,1)
C     DATA=INPUT DATA TO BE FITTED
C     *************************************************************************
C                                 GENERATE COORDINATE FUNCTIONS
            QIT=FLOAT(IT)
            DELT=(X2-X1)/FLOAT(IT-1)
C                                 ASSUME FIRST ALL ARE LEGENDRE
      DO 1  I=1,IT
            Z=X1+DELT*FLOAT(I-1)
            A(I,1)=1.0
      GO TO (14,15),INTVAL
 14         A(I,2)=2.0*Z-1.0
      GO TO 16
 15         A(I,2)=Z
 16   DO 2  J=3,JT
            M=J-1
            A(I,J)=(FLOAT(2*M-1)*A(I,2)*A(I,J-1)-FLOAT(M-1)*
     1      A(I,J-2))/FLOAT(M)
```

```
 2    CONTINUE
C                                 REPLACE THE FOLLOWING LEGENDRE TERMS
C                                 WITH SPECIAL TERMS
      DO 3  M=1,3
              IF(K(M))     3,3,4
 4            LL=L(M)
              KK=K(M)
              A(I,KK)=Z**LL
 3    CONTINUE
 1    CONTINUE
C                                 FORM NORMAL EQUATIONS
      DO 8  M=1,JT
      DO 9  N=1,M
              D(M,N)=0.0
              UNIT(M,N)=0.0
              UNIT(N,M)=0.0
      DO 10  I=1,IT
              D(M,N)=D(M,N)+A(I,M)*A(I,N)
 10   CONTINUE
      IF(M-N)  11,17,11
 11           D(N,M)=D(M,N)
      GO TO 9
 17   UNIT(M,N)=1.0
 9    CONTINUE
 8    CONTINUE
      DO 12  M=1,MT
      DO 12  N=1,JT
              E(M,N)=0.0
      DO 12  I=1,IT
              E(M,N)=E(M,N)+A(I,N)*DATA(I,M)
 12   CONTINUE
C     GET INVERSE
              DET=1.0
              MOUSE=ISIMEQ(20,JT,JT,D,UNIT,DET,SPACE)
      GO TO (5,6,6),MOUSE
 6    PRINT 100
```

```
  100 FORMAT(20H     ERROR IN ISIMEQ )
      CALL EXIT
C     FIND COEFFICIENTS
 5    DO 13  M=1,MT
      DO 13  N=1,JT
                C(N,M)=0.0
      DO 13  J=1,JT
                C(N,M)=C(N,M)+D(N,J)*E(M,J)
 13   CONTINUE
C                                      EVALUATE POLYNOMIAL AT INPUT POINTS
      DO 18  M=1,MT
                R(M)=0.0
      DO 19  I=1,IT
                H(I,M)=0.0
      DO 7  J=1,JT
                H(I,M)=H(I,M)+C(J,M)*A(I,J)
 7    CONTINUE
C                                      COMPUTE ERROR
                R(M)=R(M)+(H(I,M)-DATA(I,M))**2
 19   CONTINUE
                R(M)=SQRT(R(M))/QIT
 18   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LEGTR(K,INTVAL,A)
      DIMENSICN A(20,20)
C     THIS SUBROUTINE CCMPUTES A K BY K TRANSFCRMATICN MAR
C     THIS SUBROUTINE COMPUTES A K BY K TRANSFORMATION MATRIX, A , TO
C     CONVERT LEGENDRE PCLYNCMIALS
C     CONVERT LEGENDRE POLYNOMIAL COEFFICIENTS TO ORDINARY ONES.
C     IF INTVAL=1 THE INTERVAL IS (0.1) . IF UNTVAL=2 THE INTERVAL IS (-1,1)
      DO 1 M=1,K
      DO 1 N=1,K
                A(M,N)=0.0
    1 CONTINUE
                A(1,1)=1.0
      GO TO (2,3),INTVAL
    2           A(2,1)=-1.0
                A(2,2)=2.0
      GO TO 4
    3           A(2,2)=1.0
    4 DO 5 M=3,K
                B=M-1
                C=M-2
                D=2*M-3
                P=C/B
      GO TO (6,7),INTVAL
    6           Q=D/B
                R=2.0*Q
       GO TO 8
    7           Q=0.0
                R=D/B
    8           JT=M-1
      DO 5 J=1,JT
                A(M,J)=A(M,J)-P*A(M-2,J)-Q*A(M-1,J)
                A(M,J+1)=A(M,J+1)+R*A(M-1,J)
    5 CONTINUE
      RETURN
      END
```

```
      FUNCTION ISIMEQ ( DSM , NE , NC , A , B , DET , C )
C     SUBPPROGRAM TO SOLVE SIMULTANEOUS LINEAR EQUATIONS
C       ARGUMENTS -
C          DSM      DIMENSIONED SIZE OF COEFFICIENT MATRIX
C          NE       ACTUAL NUMBER OF EQUATIONS FOR THIS CALL
C          NC       NUMBER OF COLUMNS IN CONSTANT MATRIX
C          A        COEFFICIENT MATRIX
C          B        CONSTANT MATRIX
C          DET      INPUT - SCALE FACTOR - OUTPUT - FACTOR TIMES
C                   DETERMINANT VALUE OF COEFFICIENT MATRIX
C          C        TEMPORARY STORAGE FOR SUBROUTINE
C          ISIMEQ  RETURNS 1 IF OK. 2 IF OVRFLO. 3 IF SINGULAR
C          IF NC IS NEGATIVE, THE INVERSE OF THE COEFFICIENT MATRIX I
C          S REQUIRED, MATRIX B IS SET UP AS IDENTITY
      LOGICAL OVO
      INTEGER DSM,C,T,SUB1,SUB2,R,D
      DIMENSION A(1), B(1), C(1)
C     INITIALIZE
      N=NE
      D=DSM
      M=IABS(NC)
      ISIMEQ=1
      OVO=.FALSE.
      DO 1 I=1,N
 1    C(I)=I
      IF(NC) 5,15,15
C     INVERSE REQUIRED
 5    SUB2=0
      DO 10 J=1,N
      SUB1=SUB2
      DO 6 I=1,N
      SUB1=SUB1+1
 6    B(SUB1)=0.0
      SUB1=SUB2+J
      B(SUB1)=1.0
 10   SUB2=SUB2+D
```

```
C     START MAIN LOOP
 15   DO 1000 L=1,N
      LP1=L+1
      DO 40 I=L,N
      PIVOT=0.0
      SUB1=(L-1) * D + I
      SUB2=SUB1
      DO 20 J = L , N
      IF ( ABS(PIVOT) .GE. ABS(A(SUB1)))  GO TO 20
      PIVOT = A(SUB1)
      JB=J
 20   SUB1 = SUB1 + D
C     COMPUTE DETERMINANT
 21   DET = DET * PIVOT
 24   IF(PIVOT .EQ. 0.0) GO TO 2000
      DO 25 J=L,N
      A(SUB2) = A(SUB2) / PIVOT
 25   SUB2 = SUB2 + D
      IF (DVO) GO TO 35
      SUB1 = I
      DO 30 J = 1,M
      B(SUB1) = B(SUB1) / PIVOT
 30   SUB1 = SUB1 + D
 35   IF ( I .EQ. L) JP= JB
 40   CONTINUE
C     INTERCHANGE COLUMNS
 100  IF (JP .EQ. L) GO TO 260
      IF (DVO) GO TO 110
      T= C(L)
      C(L)=C(JP)
      C(JP) = T
 110  R = D * L - D
      T = D * JP - D
      DO 120 I=1,N
      SUB1 = R + I
      SUB2 = T + I
```

```
      S = A(SUB1)
      A(SUB1) = A(SUB2)
  120 A(SUB2) = S
      DET = -DET
C     REDUCE PIVOT COLUMN
  260 R = D * L - D
      DO 400 I=1,N
      IP = R + I
      PIVOT= A(IP)
      IF (I .EQ. L .OR.PIVOT .EQ. 0.0) GO TO 400
      SUB1 = L
      SUB2 = I
      DO 360 J = 1,N
      IF ( J .LT. LP1) GO TO 300
      S = PIVOT * A(SUB1)
      A(SUB2) = A(SUB2) - S
      IF (ABS(A(SUB2)) .LT. ABS(3.0E-8 * S)) A(SUB2) = 0.0
  300 IF(DVO .OR. J .GT. M) GO TO 350
      B(SUB2) = B(SUB2) - PIVOT * B(SUB1)
  350 SUB1 = SUB1 + D
  360 SUB2 = SUB2 + D
  400 CONTINUE
 1000 CONTINUE
      IF (DVO) GO TO 1500
C     REARRANGE VARIABLES
 1100 DO 1200 L = 1,N
      SUB1 = C(L)
      SUB2 = L
      DO 1200 J= 1,M
      A(SUB1) = B(SUB2)
      SUB1 = SUB1 + D
 1200 SUB2 = SUB2 + D
 1500 RETURN
C     SINGULAR COEFFICIENT MATRIX
 2000 ISIMEQ= 3
      GO TO 1500
```

END

VALUES OF THE HYDROSTATIC FORCE AND MOMENT
AS A FUNCTION OF HEAVE AND ROLL DISPLACEMENTS
DISCRIPTION OF PARAMETERS

Z/(RO*G*B**3) - NONDIMENSIONAL FORCE
M/(RO*G*B**4) - NONDIMENSIONAL MOMENT
Z(O)/(T/2) - NONDIMENSIONAL HEAVE DISPL.
FI - NONDIMENSIONAL ROLL DISPL.
RO - WATER DENSITY
G - GRAVITATIONAL ACCELERATION
B BEAM OF THE MODEL
T - DRAFT OF THE MODEL

1 - HEAVE FORCE

ABS(FI) \ Z(O)/(T/2)	-1.0	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8	1.0
.0	.79	.64	.49	.34	.17	0	-.18	-.37	-.52	-.71	-.90
.175	.77	.62	.48	.32	.16	-.01	-.18	-.36	-.54	-.73	-.92
.350	.72	.58	.44	.29	.14	-.03	-.20	-.35	-.53	-.70	-.87
.525	.64	.51	.38	.24	.10	-.06	-.20	-.34	-.50	-.63	-.76
.700	.52	.41	.30	.18	.05	-.07	-.19	-.30	-.42	-.52	-.62
.875	.36	.28	.20	.10	.0	-.07	-.15	-.23	-.31	-.37	-.43
1.000	.18	.14	.12	.04	-.02	-.06	-.11	-.16	-.20	-.24	-.26

2 - ROLL MOMENT*(E01) (FOR NEG. ANGLE, MOM. IS POS.)

Z(O)/(T/2)

FI	-1.0	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8	1.0
0	0	0	0	0	0	0	0	0	0	0	0
.175	-.16	-.12	-.10	-.11	-.12	-.15	-.18	-.22	-.27	-.33	-.39
.350	-.28	-.26	-.26	-.26	-.28	-.34	-.41	-.49	-.56	-.58	-.59
.525	-.38	-.39	-.42	-.48	-.55	-.63	-.69	-.73	-.73	-.68	-.64
.700	-.44	-.52	-.61	-.70	-.77	-.82	-.84	-.84	-.81	-.74	-.66
.875	-.56	-.66	-.74	-.82	-.88	-.92	-.93	-.91	-.86	-.77	-.66
1.000	-.67	-.74	-.84	-.							
1.000	-.67	-.74	-.80	-.86	-.93	-.97	-.98	-.94	-.88	-.78	-.66

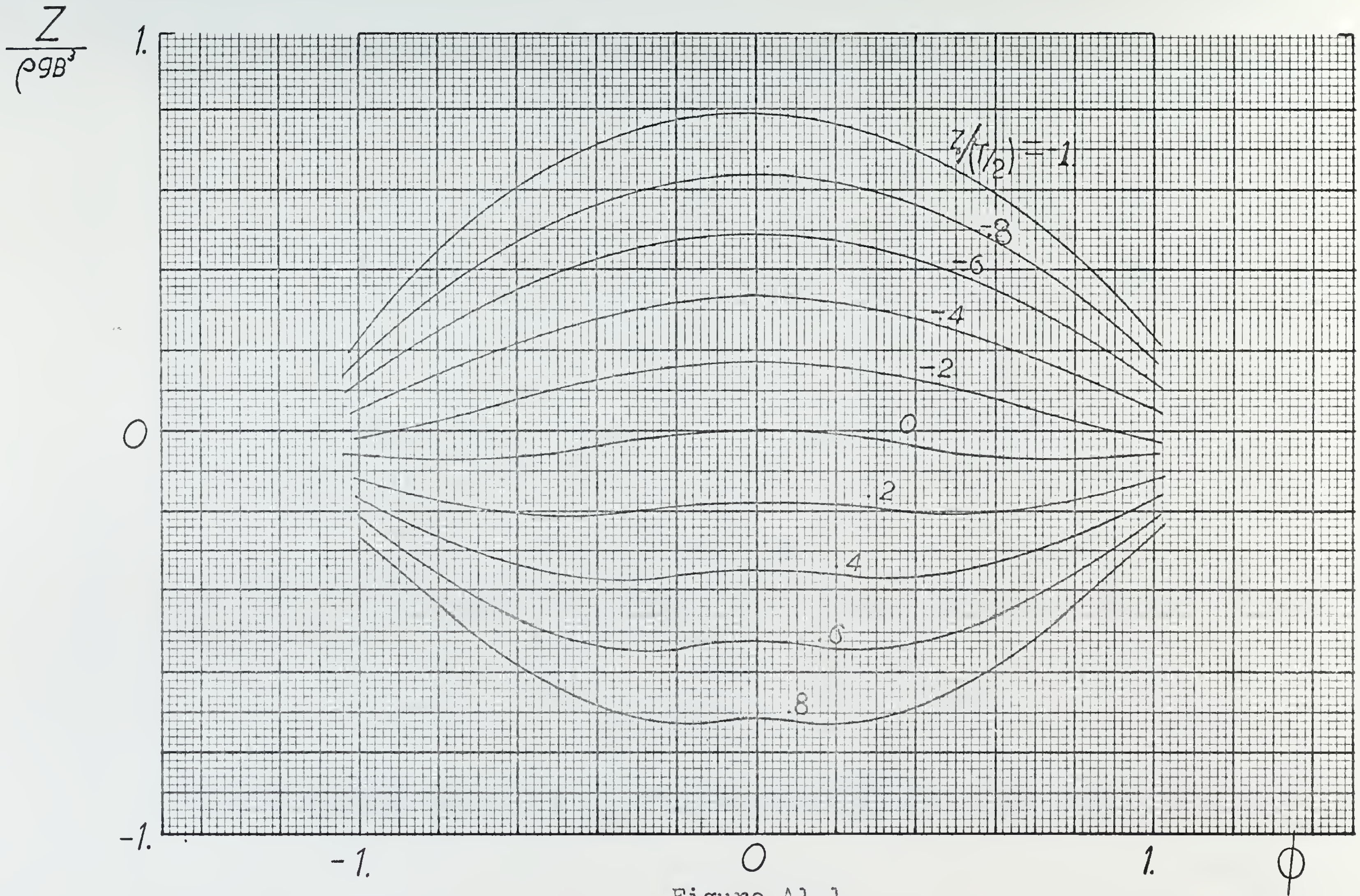

Figure A1.1

Hydrostatic force vs. angle of rotation and vertical position

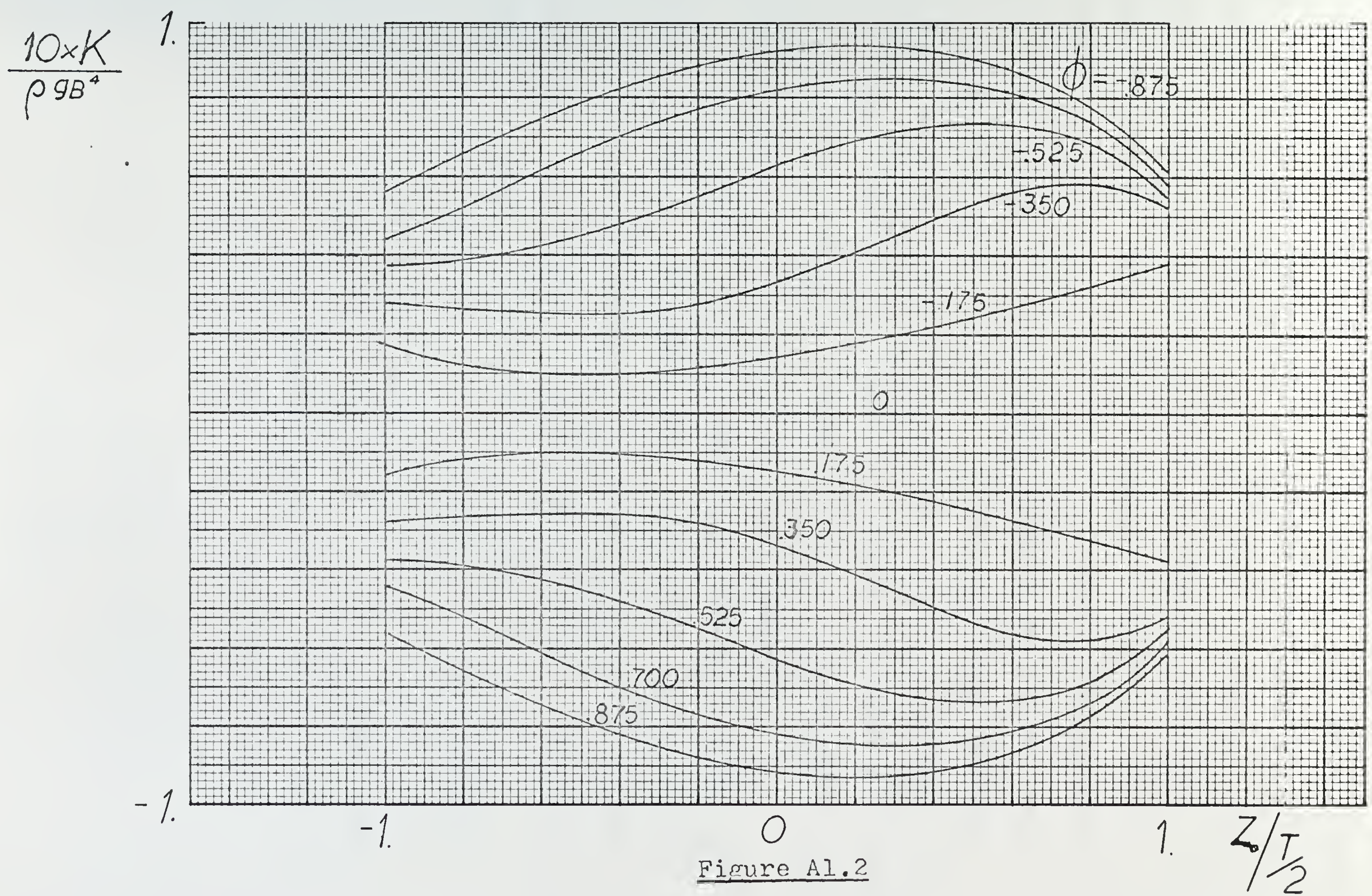

Figure A1.2

Hydrostatic moment vs. angle of rotation and vertical position

VALUES OF THE HYDROSTATIC COEFFICIENTS

Since the identification of the motion parameters in coupled heave and roll was not performed, it was necessary to calculate the hydrostatic coupling coefficients. To calculate only the coefficients Z_z , Z_z2, K_ϕ and $K_\phi3$ a much simpler method than resorting to the Kerwin polynomial representation and that will probabilly yeld more accurate results can be used . The reason for such is that in computerized solutions using least squares, it is not usually possible to specify some a priori knowledge that one might have about the surface or the curve. For example in roll $K(\phi=0)$ and $K_\phi2\Big|_{\phi=0}$ are zero due to port and starboard symmetry. Also $K_\phi\Big|_{\phi=0} = -$ W*GM . A least square method gave nonzero values for the first two coefficientes.

Looking just for the secon non zero term in the Taylor's expansion and knowing the first it's much easier to calculate manually the missing term. Using this procedure and being, for roll K_o - W*GM -2.159 LB*FT, the value that will approximate the moment curve better will be :

Between	$\phi=\pm.175$	$1/3'*K_o3$	= 6.814 LB*FT
	$\phi=\pm.350$		= -1.254 LB*FT
	$\phi=\pm.525$		= -3.007 LB*FT

Since the roll angle will go over .175 but does not reach .350 in the experiments the value choosen was:

$1/3'K_o3$ = 6.814 LB*FT

Similarly for heave the values found were

$Z_z = -\rho*g*A_{1w}$ = -194.709 LB/FT

$1/2'Z_z2$ = - 84.528 LB/FT^2

APPENDIX 2

LINEAR HYDRODYNAMIC COEFFICIENTS

1 - DISCRIPTION OF THE COMPUTER PROGRAMS TO CALCULATE ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENTS FOR TWO DIMENSIONAL CYLINDERS IN HEAVE, SWAY AND ROLL

2 - VALUES OF ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENTS FOR A MODEL OF THE " MARINER " HULL FORM IN HEAVE AND ROLL

PROGRAM INPUT

Cards 1-4 FORMAT(20A4)

The purpose of these cards is to provide titles for the computed output. When titles are not desired blank cards should be provided.

Card 5 -NOK,MAXB,NH FORMAT(3I6)

NOK - Total number of frequencies, at which the calculations will be performed.(MAX 50)

MAXB -Integer correspondent to the horizontal plane of maximum beam. Start the count at the keel.

NH - Number of depths at which the calculations will be performed.(MAX 10)

Card 6 - CAY(K),K 1,NOK FORMAT(5F12.7)

CAY - Nondimensional frequencies - a/g

- radian frequency

a - half the beam for a floating cylinder

- half the draft for a submerged cylinder

g - gravitational acceleration

Card 7 -(DEP(J),J 1,NH),CR FORMAT(5F12.7)

DEP - Values of the depths of submergence, measured from the surface to the top of the cylinder. For a floating cylinder, NH 1 and DEP 0 .

Card 8 - MD,NON FORMAT(2I6)

MD - Control integer - 1 for a submerged body

2 for a floating body

NON - Number of segments in which the right hand side half contour is divided (MAX 45)

CR - Positive vertical distance from the center of rotation to the free surface, for a floating cylinder or from the center of rotation to the intersection of the section contour with the plane of symmetry(on the side closer to the surface.

Card 9 - X(J),J 1, (NON 1) FORMAT(5F12.7)

X(J) - Horizontal offsets of the end points of the line segments, measured from the vertical planr of symmetry to the right hand side of the contour, starting at the keel. The last point is the intersection of the section contour either with the free surface (floating cylinder) or with the vertical plane of symmetry(submerged cylinder)

Card 10 - Y(J),J 1, (NON 1) FORMAT(5F12.7)

Y(J) - Negative vertical offsets of the end points of the line segments, measured from th free surface, for a floating cylinder or from the intersection of the contour with the vertical plane of symmetry, for a submerged cylinder, starting from the keel. Again only the right hand side of the contour will be considered.

Card 11 - MODE FORMAT(I6)

MODE - Control integer - 1 - heave

2 - sway

3 - roll

Card 12 - FORMAT(20A4)

Provides a title for the output of the program. If the title is not desired, a blank card should be provided.

If more than one mode is desired, adicional pairs of cards 11 and 12 should be provided, accordingly.

Card 13 - MODE FORMAT(I6)

MODE - Control variable

0 - End of data

Negative integer - A new geometry follows starting Card 5.

LISTINGS OF THE PROGRAMS

THE FOLLOWING PROGRAMS WERE TAKEN FROM
NSRDC REPORT 3551
" NUMERICAL CALCULATION OF ADDED MASS AND DAMPING
COEFFICIENTS OF CYLINDERS OSCILLATING IN OR BELOW
A FREE SURFACE "

BY
J.W. BEDEL AND C.M. LEE

```
C     MAIN  PROGRAM  --  YFA4
      DIMENSION XA(46),YA(46)
      COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CK,RAT,SUK,DEG,JERK,DRT,HBM,SG,N
     1OE,PDM,VOL,DEW,ON,OMEGA,CP,WVH,ID,DOG,IG,SEN(46),CES(46),XX(45),YY
     2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90
     3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45),DEP(10)
      COMMON/GR/NOK,NOT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
     1(46),Y(46)
      COMMON/FQ/K
      COMMON/TT/TEST
      COMMON/GRPH/TITD(20),DET(50),LAD,LDT,TITA(20),TITV(20),PAK(50),PVK
     1(50),PAS(50),PVS(50),LPV
      COMMON/SHP/MAXB,DUL(45)
      COMMON/MOD/F(5),D(5)
    1 FORMAT(12I6)
    2 FORMAT(5F12.7)
    3 FORMAT(20A4)
    4 FORMAT(////64H       SUBMERGED CYLINDERS OSCILLATING UNDER THE FREE
     1 SURFACE//6X,40H H/D = DEPTH TO TOP OF BODY / HALF DRAFT)
```

```
    5 FORMAT(39H0     TRANSFORMATION NOT DEFINED - STOP)
    6 FORMAT(24H0     MATRIX IS SINGULAR)
    7 FORMAT(34H0     HEAVING OSCILLATIONS, H/D = F10.5)
    8 FORMAT(34H0     SWAYING OSCILLATIONS, H/D = F10.5)
    9 FORMAT(33H0     ROLLING OSCILLATIONS ABOUT F10.5,8H  H/D = F10.5)
   10 FORMAT(12H0     CAY = F8.4,13H      NONWL = F8.4)
   11 FORMAT(12H0     AMC = F10.5,9H   CFC = F10.5,9H   WVH = F10.5)
   12 FORMAT(42H0     PRESSURES IN PHASE WITH ACCELERATION//)
   13 FORMAT(5F12.5)
   14 FORMAT(33H0     PRESSURES IN PHASE WITH VELOCITY//)
   29 FORMAT(29H0     DETERMINANT = E15.8)
   34 FORMAT(6X,14HSTRUT DEPTH = F4.2,10H,    AFC = F8.4,8H DEGREES/6X,20
     1HDFC = 0.0 FOR CAY = F4.2)
   35 FORMAT(1H0,12X,12HHALF BEAM = F10.5/17X,8HDRAFT = F10.5/6X,19HAREA
     1 COEFFICIENT = F10.5)
   36 FORMAT(1H0,5X,36HABSCISSAS OF CYLINDRICAL CROSS SECTION//)
   37 FORMAT(1H0,5X,38HORDINATES OF CYLINDRICAL CROSS SECTION//)
   42 FORMAT(1H0,5X,42HIMPULSIVE SURFACE CONDITION, NU VERY LARGE)
   52 FORMAT(////61H       SEMISUBMERGED CYLINDERS OSCILLATING IN THE FRE
     1E SURFACE)
   54 FORMAT(5H MD =,I3,4X,                     6H NON =,I3,4X,6H NOK =,I3,4X
     1,5H CR =,F9.4)
   65 FORMAT(2H1 ,20A4)
   66 FORMAT(///2X,11H CAY VALUES//(5F12.7))
   67 FORMAT(/// 2X,13H INPUT VALUES///)
   68 FORMAT(//2X,7H DEPTHS//(5F12.7))
   69 FORMAT(2H1 ,20A4//)
   70 FORMAT(2X,////)
   71 FORMAT(2H1 )
      PI=3.1415927
      HPI=.5*PI
      QPI=.5*HPI
      TPI=2.*PI
      F(1)=0.52175561
      F(2)=0.39866681
      F(3)=0.07594245
```

```
      F(4)=0.0003611758
      F(5)=0.000023369972
      D(1)=0.26350032
      D(2)=1.4134031
      D(3)=3.5904258
      D(4)=7.0085100
      D(5)=12.645001
      READ(5,3)(TITU(J),J=1,20)
      READ(5,3)(TITD(J),J=1,20)
      READ(5,3)(TITA(J),J=1,20)
      READ(5,3)(TITV(J),J=1,20)
      WRITE(6,65)(TITU(J),J=1,20)
      WRITE(6,3)(TITD(J),J=1,20)
      WRITE(6,3)(TITA(J),J=1,20)
      WRITE(6,3)(TITV(J),J=1,20)
   55 READ(5,1) NOK,MAXB,NH
      READ(5,2)(CAY(K),K=1,NOK)
      READ(5,2)(DEP(J),J=1,NH),CR
      READ(5,1) MD,NON
      NUT=NON+1
      READ(5,2) (XA(J),J=1,NUT)
      READ(5,2) (YA(J),J=1,NUT)
      HBEAM=XA(MAXB)
   53 AREA=0.0
      DO 54 J=1,NON
      XX(J)=.5*(XA(J)+XA(J+1))
      XINT=XA(J+1)-XA(J)
      YINT=YA(J+1)-YA(J)
      DA=YINT*XX(J)
      AREA=AREA+DA
      DUL(J)=SQRT(XINT**2+YINT**2)
      SNE(J)=YINT/DUL(J)
   54 CSE(J)=XINT/DUL(J)
      DRAFT=YA(NUT)-YA(1)
      AREA=AREA/(HBEAM*DRAFT)
      GO TO (56,51),MD
```

```
51 WRITE(6,52)
   GO TO 53
50 WRITE(6,4)
53 WRITE(6,67)
   WRITE(6,54) MD,NCN,NOK,CR
   WRITE(6,66) (CAY(K),K=1,NOK)
   WRITE(6,68) (DEP(J),J=1,NH)
   WRITE(6,56)
   WRITE(6,13) (XA(J),J=1,NUT)
   WRITE(6,57)
   WRITE(6,13) (YA(J),J=1,NUT)
   WRITE(6,35) HBEAM,DRAFT,AREA
38 READ(5,1) MODE
   IF(MODE) 40,40,41
41 READ(5,3)(TITLE(J),J=1,20)
   DO 20 I5=1,NH
   WRITE(6,69)(TITLE(J),J=1,20)
   DPH=DEP(I5)
   DO 78 J=1,NUT
   X(J)=XA(J)
78 Y(J)=YA(J)
17 CALL SHAPED
18 CALL FIND
32 GO TO(21,22,23),MODE
21 WRITE(6,7)DPH
   GO TO 24
22 WRITE(6,8)DPH
   GO TO 24
23 WRITE(6,9)CR,DPH
24 WRITE(6,70)
   DO 20 K=1,NOK
   UN=CAY(K)/PDM
   OMEGA=SQRT(ABS(UN))
   WLN=2.*PI/ABS(CAY(K))
   WN=CAY(K)/DRT
   IF(UN)43,44,44
```

```
43 WRITE(6,42)
   GO TO 45
44 WRITE(6,10) CAY(K),WLN
45 CALL FREQ
   ID=ID
   GO TO(25,28),ID
28 WRITE(6,6)
   GO TO 48
25 WRITE(6,11)AMC(K),DFC(K),WVH
   WRITE(6,12)
   WRITE(6,13)(PRA(J),J=1,NCN)
   WRITE(6,14)
   WRITE(6,15)(PRV(J),J=1,NCN)
   WRITE(6,29)DUG
   PAK(K)=PRA(1)
   PVK(K)=PRV(1)
   PAS(K)=PRA(NCN)
   PVS(K)=PRV(NCN)
   IF(UM)46,46,47
46 DET(K)=0.0
   GO TO 48
47 DET(K)=DUG
48 WRITE(6,70)
26 CONTINUE
27 GO TO 38
40 WRITE(6,71)
   IF(MODE.LT.0) GO TO 55
   STOP
   END
```

```
      SUBROUTINE SHAPED
C     CALCULATION OF GEOMETRIC PARAMETERS
      COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,KAT,SUR,DEG,JERK,DRT,HBM,SG,N
     1OE,PDM,VOL,DEW,UN,OMEGA,CP,WVH,ID,DUG,IG,SEN(46),CES(46),XX(45),YY
     2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90
     3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)
      COMMON/GR/NOK,NUT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
     1(46),Y(46)
      COMMON/SHP/MAXB,DUL(45)
      JERK=1
      KAB=MAXB
      GO TO (10,15),MD
   15 D=X(KAB)
      DPH=0.0
      GO TO 53
   10 D=.5*(Y(NUT)-Y(1))
   53 DO 54 J=1,NUT
      X(J)=X(J)/D
   54 Y(J)=Y(J)/D
      DRT=D
      HBM=X(KAB)
      CP=CR/D
      IF(DPH) 24,24,21
   21 DPH=DPH/D
      DO 22 J=1,NUT
   22 Y(J)=Y(J)-DPH
      NUT=NON+1
   24 DO 25 J=1,NON
      XX(J)=.5*(X(J)+X(J+1))
      YY(J)=.5*(Y(J)+Y(J+1))
   25 DEL(J)=DUL(J)/D
      SG=SING(MODE)
      NOE=2*NON
      GO TO(26,27,28),MODE
   26 DO 29 J=1,NON
   29 FR(J)=CSE(J)
```

```
33 PDM=1.0
   DEN=1.
   GO TO (31,30),MD
30 VOL=HPI
   GO TO 37
31 VOL=PI
   GO TO 37
27 DO 32 J=1,NUN
32 FR(J)=-SNE(J)
   GO TO 33
28 DO 34 J=1,NUN
34 FR(J)=(YY(J)-CP)*SNE(J)+XX(J)*CSE(J)
   DEN=HBM
   PDM=1.0
   GO TO (36,35),MD
35 VOL=QPI
   GO TO 37
36 VOL=HPI
37 RETURN
   END
```

```
      SU296UT955 0954                                                43  0
C     CALCULATION OF THE NORMAL DERIVATIVES OF THE LOGARITHMIC SINGULARITIES
      COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,RAT,SUR,DEG,JERK,DRT,HBM,SG,NFIND0020
     1JE,PDM,VOL,DEW,UN,OMEGA,CP,WVH,ID,DUG,IG,SEN(46),CES(46),XX(45),YYFIND0030
     2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90FIND0040
     3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)             FIND0050
      COMMON/GR/NOK,NUT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
     1(46),Y(46)                                                        FIND0070
      COMMON/SHP/MAXB,DUL(45)
      DO 1 I=1,NON                                                      FIND0080
      XM1=XX(I)-X(1)                                                    FIND0090
      YM1=YY(I)-Y(1)                                                    FIND0100
      XP1=XX(I)+X(1)                                                    FIND0110
      YP1=YY(I)+Y(1)                                                    FIND0120
      FPR1=.5*ALOG(XM1**2+YM1**2)                                       FIND0130
      FPL1=.5*ALOG(XP1**2+YM1**2)                                       FIND0140
      FCR1=.5*ALOG(XM1**2+YP1**2)                                       FIND0150
      FCL1=.5*ALOG(XP1**2+YP1**2)                                       FIND0160
      APR1=ATAN2(YM1,XM1)
      APL1=ATAN2(YM1,XP1)
      ACR1=ATAN2(YP1,XM1)
      ACL1=ATAN2(YP1,XP1)
      DO 1 J=1,NON                                                      FIND0210
      XM2=XX(I)-X(J+1)                                                  FIND0220
      YM2=YY(I)-Y(J+1)                                                  FIND0230
      XP2=XX(I)+X(J+1)                                                  FIND0240
      YP2=YY(I)+Y(J+1)                                                  FIND0250
      FPR2=.5*ALOG(XM2**2+YM2**2)                                       FIND0260
      FPL2=.5*ALOG(XP2**2+YM2**2)                                       FIND0270
      FCR2=.5*ALOG(XM2**2+YP2**2)                                       FIND0280
      FCL2=.5*ALOG(XP2**2+YP2**2)                                       FIND0290
      APR2=ATAN2(YM2,XM2)                                               FIND0300
      APL2=ATAN2(YM2,XP2)                                               FIND0310
      ACR2=ATAN2(YP2,XM2)                                               FIND0320
      ACL2=ATAN2(YP2,XP2)                                               FIND0330
      SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)                                  FIND0340
```

```
      CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)                                  FIND0350
      SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)                                  FIND0360
      CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)                                  FIND0370
      DPNR=SIMJ*(FPR1-FPR2)+CIMJ*(APR1-APR2)                            FIND0380
   99 PPR=CSE(J)*(XM1*FPR1-YM1*APR1-XM1-XM2*FPR2+YM2*APR2+XM2)+SNE(J)*(YFIND0390
     1M1*FPR1+XM1*APR1-YM1-YM2*FPR2-XM2*APR2+YM2)                       FIND0400
      DPNL=SIPJ*(FPL2-FPL1)+CIPJ*(APL2-APL1)                            FIND0410
      PPL=CSE(J)*(XP2*FPL2-YM2*APL2-XP2-XP1*FPL1+YM1*APL1+XP1)+SNE(J)*(YFIND0420
     1M1*FPL1+XP1*APL1+YM2-YM2*FPL2-XP2*APL2-YM1)                       FIND0430
      DCNR=SIPJ*(FCR1-FCR2)+CIPJ*(ACR1-ACR2)                            FIND0440
      PCR=CSE(J)*(XM1*FCR1-YP1*ACR1-XM1-XM2*FCR2+YP2*ACR2+XM2)+SNE(J)*(YFIND0450
     1P2*FCR2+XM2*ACR2+YP1-YP1*FCR1-XM1*ACR1-YP2)                       FIND0460
      DCNL=SIMJ*(FCL2-FCL1)+CIMJ*(ACL2-ACL1)                            FIND0470
      PCL=CSE(J)*(XP2*FCL2-YP2*ACL2-XP2-XP1*FCL1+YP1*ACL1+XP1)+SNE(J)*(YFIND0480
     1P2*FCL2+XP2*ACL2-YP2-YP1*FCL1-XP1*ACL1+YP1)                       FIND0490
      BLOG(I,J)=DPNR+SG*DPNL-DCNR-SG*DCNL                               FIND0500
      YLOG(I,J)=PPR+SG*PPL-PCR-SG*PCL                                   FIND0510
      IF(J-NON)2,1,1                                                    FIND0520
    2 XM1=XM2                                                           FIND0530
      YM1=YM2                                                           FIND0540
      XP1=XP2                                                           FIND0550
      YP1=YP2                                                           FIND0560
      FPR1=FPR2                                                         FIND0570
      FPL1=FPL2                                                         FIND0580
      FCR1=FCR2                                                         FIND0590
      FCL1=FCL2                                                         FIND0600
      APR1=APR2                                                         FIND0610
      APL1=APL2                                                         FIND0620
      ACR1=ACR2                                                         FIND0630
      ACL1=ACL2                                                         FIND0640
    1 CONTINUE                                                          FIND0650
      RETURN                                                            FIND0660
      END                                                               FIND0670
```

```
      SUBROUTINE DAVID(X,Y,E,C,S,RA,RB,CIN,SON)
C     COMPUTATION OF EXPONENTIAL INTEGRAL WITH COMPLEX ARGUMENT
      COMMON/MOD/F(5),D(5)
      Q=3.1415927
      AT=ATAN2(X,Y)
      ARG=AT-.5*Q
      E=EXP(-Y)
      C=COS(X)
      S=SIN(X)
      R=X**2+Y**2
      AL=0.5*ALOG(R)
      A=-Y
      B=-X
      IF(A.GE.0.0) GO TO 78
      IF(B.EQ.0.0) GO TO 79
   78 IF(R.GE.100.) GO TO 10
   79 TEST=0.00001
      IF(R.LT.1.0) GO TO 5
      TEST=0.1*TEST
      IF(R.LT.2.0) GO TO 5
      TEST=0.1*TEST
      IF(R.LT.4.0) GO TO 5
      TEST=0.1*TEST
    5 CONTINUE
      SUMC=0.57721566+AL+Y
      SUMS=AT+X
      TC=Y
      TS=X
      DO 1 K=1,500
      TJ=TC
      COX=K
      CAY=K+1
      FACT=COX/CAY**2
      TC=FACT*(Y*TC-X*TS)
      TS=FACT*(Y*TS+X*TJ)
      SUMC=SUMC+TC
```

```
      SUMS=SUMS+TS
      IF(K.GE.500) GO TO 3
      IF((ABS(TC)+ABS(TS)).GT.TEST) GO TO 1
    3 CIN=E*(C*SUMC+S*SUMS)
      SUN=E*(S*SUMC-C*SUMS)
      GO TO 4
    1 CONTINUE
   10 G1=0.
      G2=0.
      DO 20 I=1,5
      DEN=(-Y+D(I))**2+X**2
      GA=F(I)*(-Y+D(I))/DEN
      GB=F(I)*(-X)/DEN
      G1=G1+GA
   20 G2=G2+GB
      CIN=E*Q*S-G1
      SUN=-(E*Q*C+G2)
    4 RA=AL-CIN
      RB=ARG+SUN
      RETURN
      END
```

```
      FUNCTION SING(N)
C     ASSIGNEMENT OF POSITIVE OR NEGATIVE SIGNAL, DEPENDING ON THE MODE OF
C     MOTION
      IF(1-N)2,1,1
    1 SING=1.
      GO TO 77
    2 SING=-1.
   77 RETURN
      END
```

```
      SUBROUTINE FREQ
C     CALCULATION OF THE SOURCE STRENGHT ON THE CONTOUR SEGMENTS
      COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,RAT,SUR,DEG,JERK,DRT,HBM,SG,N
     1DE,PDM,VOL,DEW,UN,OMEGA,CP,WVH,ID,DUG,IG,SEN(46),CES(46),XX(45),YY
     2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90
     3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)
      COMMON/GR/NOK,NUT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
     .1(46),Y(46)
      COMMON/FQ/K
      COMMON/IT/TEST
      IF(UN)8,9,10
    8 DO 11 I=1,NON
      DO 11 J=1,NON
      CT(I,J)=BLOG(I,J)
   11 PSI1(I,J)=YLOG(I,J)
      GO TO 14
    9 DO 12 I=1,NON
      XM1=XX(I)-X(1)
      XP1=XX(I)+X(1)
      YP1=YY(I)+Y(1)
      FCR1=.5*ALOG(XM1**2+YP1**2)
      FCL1=.5*ALOG(XP1**2+YP1**2)
      ACR1=ATAN2(YP1,XM1)
      ACL1=ATAN2(YP1,XP1)
      DO 12 J=1,NON
      XM2=XX(I)-X(J+1)
      XP2=XX(I)+X(J+1)
      YP2=YY(I)+Y(J+1)
      FCR2=.5*ALOG(XM2**2+YP2**2)
      FCL2=.5*ALOG(XP2**2+YP2**2)
      ACR2=ATAN2(YP2,XM2)
      ACL2=ATAN2(YP2,XP2)
      SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)
      CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)
      SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)
      CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)
```

```
      DCNR=SIPJ*(FCR1-FCR2)+CIPJ*(ACR1-ACR2)
      PCR=CSE(J)*(XM1*FCR1-YP1*ACR1-XM1-XM2*FCR2+YP2*ACR2+XM2)+SNE(J)*(Y
     1P2*FCR2+XM2*ACR2+YP1-YP1*FCR1-XM1*ACR1-YP2)
      DCNL=SIMJ*(FCL2-FCL1)+CIMJ*(ACL2-ACL1)
      PCL=CSE(J)*(XP2*FCL2-YP2*ACL2-XP2-XP1*FCL1+YP1*ACL1+XP1)+SNE(J)*(Y
     1P2*FCL2+XP2*ACL2-YP2-YP1*FCL1-XP1*ACL1+YP1)
      CT(I,J)=BLOG(I,J)+2.0*(DCNR+SG*DCNL)
      PSI1(I,J)=YLOG(I,J)+2.0*(PCR+SG*PCL)
      IF(J-NON)13,12,12
   13 XM1=XM2
      XP1=XP2
      YP1=YP2
      FCR1=FCR2
      FCL1=FCL2
      ACR1=ACR2
      ACL1=ACL2
   12 CONTINUE
   14 DO 15 I=1,NON
   15 CON(I,1)=FR(I)
      CALL MATINV(CT,NON,CON,1,DOG,ID)
      GO TO(16,6),ID
   16 DO 17 I=1,NON
      PRA(I)=0.0
      PRV(I)=0.0
      DO 17 J=1,NON
   17 PRA(I)=PRA(I)-CON(J,1)*PSI1(I,J)
      AMC(K)=0.0
      DFC(K)=0.0
      WVH=0.0
      DO 18 I=1,NON
   18 AMC(K)=AMC(K)+PRA(I)*DEL(I)*FR(I)
      AMC(K)=2.0*AMC(K)/VOL
      GO TO 6
   10 DO 1 I=1,NON
      NI=NON+I
      CON(I,1)=0.
```

```
      CON(NI,1)=OMEGA*FR(I)
      XR1=UN*(XX(I)-X(1))
      YR1=-UN*(YY(I)+Y(1))
      XL1=UN*(XX(I)+X(1))
      YL1=YR1
      CALL  DAVID(XR1,YR1,EJ1,CXR1,SXR1,RAR1,RBR1,CR1,SR1)
      CALL  DAVID(XL1,YL1,EJ1,CXL1,SXL1,RAL1,RBL1,CL1,SL1)
      DO 1  J=1,NON
      NJ=NJN+J
      XR2=UN*(XX(I)-X(J+1))
      YR2=-UN*(YY(I)+Y(J+1))
      XL2=UN*(XX(I)+X(J+1))
      YL2=YR2
      CALL  DAVID(XR2,YR2,EJ2,CXR2,SXR2,RAR2,RBR2,CR2,SR2)
      CALL  DAVID(XL2,YL2,EJ2,CXL2,SXL2,RAL2,RBL2,CL2,SL2)
      SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)
      CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)
      SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)
      CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)
   25 CT(I,J)=BLOG(I,J)+2.*(SIPJ*(CR1-CR2)-CIPJ*(SR1-SR2)-SG*(SIMJ*(CL1-
     1CL2)-CIMJ*(SL1-SL2)))
      PSI1(I,J)=YLOG(I,J)+2./UN*(SNE(J)*(RAR1-RAR2)+CSE(J)*(RBR1-RBR2)+S
     1G*(SNE(J)*(RAL1-RAL2)+CSE(J)*(RBL2-RBL1)))                        FREQ
   27 CT(NI,NJ)=CT(I,J)
      CT(I,NJ)=TPI*(EJ2*(SXR2*CIPJ-CXR2*SIPJ)-EJ1*(SXR1*CIPJ-CXR1*SIPJ)-
     1SG*(EJ2*(SXL2*CIMJ-CXL2*SIMJ)-EJ1*(SXL1*CIMJ-CXL1*SIMJ)))
      PSI2(I,J)=TPI/UN*(EJ1*(SXR1*CSE(J)-CXR1*SNE(J))-EJ2*(SXR2*CSE(J)-C
     1XR2*SNE(J))-SG*(EJ1*(SXL1*CSE(J)+CXL1*SNE(J))-EJ2*(SXL2*CSE(J)+CXL
     22*SNE(J))))
      CT(NI,J)=-CT(I,NJ)
      IF(J-NON)7,1,1
    7 XR1=XR2
      YR1=YR2
      XL1=XL2
      YL1=YL2
      EJ1=EJ2
```

```
      CR1=CR2
      SR1=SR2
      CL1=CL2
      SL1=SL2
      RAR1=RAR2
      RBR1=RBR2
      RAL1=RAL2
      RBL1=RBL2
      CXR1=CXR2
      SXR1=SXR2
      CXL1=CXL2
      SXL1=SXL2
    1 CONTINUE
      CALL MATINV(CT,NOE,CON,1,DCG,ID)
      GO TO(2,6),ID
    2 DO 3 I=1,NON
      PRA(I)=0.
      PRV(I)=0.
      DO 4 J=1,NON
      NJ=NON+J
      PRA(I)=PRA(I)+CON(J,1)*PSI2(I,J)-CON(NJ,1)*PSI1(I,J)
    4 PRV(I)=PRV(I)+CON(J,1)*PSI1(I,J)+CON(NJ,1)*PSI2(I,J)
      PRA(I)=OMEGA*PRA(I)
    3 PRV(I)=OMEGA*PRV(I)
      AMC(K)=0.0
      DFC(K)=0.0
      DO 5 I=1,NON
      AMC(K)=AMC(K)+PRA(I)*DEL(I)*FR(I)
    5 DFC(K)=DFC(K)+PRV(I)*DEL(I)*FR(I)
      AMC(K)=2.0*AMC(K)
      DFC(K)=2.0*DFC(K)
      WVH=OMEGA*SQRT(ABS(DFC(K)))/DEW
      AMC(K)=AMC(K)/(UN*VOL)
      DFC(K)=DFC(K)/(UN*VOL)
    6 RETURN
      END
```

```
      SUBROUTINE MATINV(A,N1,B,M1,DETERM,ID)
C       PIVOT METHOD                                                    MAT40001
C        MATRIX  INVERSION WITH ACCOMPANYING SOLUTION OF SIMUL. EQ.      MAT40002
C     PIVOT METHOD                                                      MAT40003
C     FORTRAN IV SINGLE PRECISION WITH ADJUSTABLE DIMENSION             MAT40004
C     FEBRUARY 1966   S GOOD  DAVID TAYLOR MODEL BASIN   AM MAT4        MAT40005
C          WHERE CALLING PROGRAM MUST INCLUDE                           MAT40006
C              DIMENSION A(      ), B(      ), INDEX(     )             MAT40007
C           N      IS THE ORDER OF A                                    MAT40009
C           M      IS THE NUMBER OF COLUMN VECTORS IN B(MAY BE 0)       MAT40010
C          DETERM WILL CONTAIN DETERMINANT ON EXIT                      MAT40011
C           ID     WILL BE SET BY ROUTINE TO 2 IF MATRIX A IS SINGULAR  MAT40012
C                  1 IF INVERSION WAS SUCCESSFUL                        MAT40013
C           A      THE INPUT MATRIX WILL BE REPLACED BY A INVERSEE      MAT40014
C           B      THE COLUMN VECTORS WILL BE REPLACED BY CORRESPONDING MAT40015
C                  SOLUTION VECTORS                                     MAT40016
C           INDEX WORKING STORAGE ARRAY                                 MAT40017
C          IF IT IS DESIRED TO SCALE THE DETERMINANT CARD      MAY BE   MAT40018
C          DELETED AND DETERM PRESET BEFORE ENTERING THE ROUTINE        MAT40019
C                                                                       MAT40020
      DIMENSION A(90,90),B(90,1),INDEX(90,3)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)         MAT40022
C      INITIALIZATION                                                   MAT40023
C                                                                       MAT40024
C                                                                       MAT40026
       N=N1                                                             MAT40027
      M=M1                                                              MAT40028
       DETERM = 1.0                                                     MAT40029
       DO 20 J=1,N                                                      MAT40030
   20 INDEX(J,3) = 0                                                    MAT40031
       DO 550 I=1,N                                                     MAT40032
C                                                                       MAT40033
C     SEARCH FOR PIVOT ELEMENT                                          MAT40034
C                                                                       MAT40035
      AMAX = 0.0                                                        MAT40036
      DO 105 J=1,N                                                      MAT40037
```

```
      IF(INDEX(J,3)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF(INDEX(K,3)-1) 80, 100, 715
   80 IF (      AMAX -ABS (A(J,K))) 85, 100, 100
   85 IROW=J
      ICOLUM =K
      AMAX = ABS (A(J,K))
  100 CONTINUE
  105 CONTINUE
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1
      INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW-ICOLUM) 140, 310, 140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 310, 310, 210
  210 DO 250 L=1, M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  310 PIVOT    =A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT
  330 A(ICOLUM,ICOLUM)=1.0
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
      IF(M) 380, 380, 360
  360 DO 370 L=1,M
```

```
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT                                MAT40074
C                                                                  MAT40075
C     REDUCE NON-PIVOT ROWS                                        MAT40076
C                                                                  MAT40077
  380 DO 550 L1=1,N                                                MAT40078
      IF(L1-ICOLUM) 400, 550, 400                                  MAT40079
  400 T=A(L1,ICOLUM)                                               MAT40080
      A(L1,ICOLUM)=0.0                                             MAT40081
      DO 450 L=1,N                                                 MAT40082
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T                                MAT40083
      IF(M) 550, 550, 460                                          MAT40084
  460 DO 500 L=1,M                                                 MAT40085
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T                                MAT40086
  550 CONTINUE                                                     MAT40087
C                                                                  MAT40088
C     INTERCHANGE COLUMNS                                          MAT40089
C                                                                  MAT40090
      DO 710 I=1,N                                                 MAT40091
      L=N+1-I                                                      MAT40092
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630                     MAT40093
  630 JROW=INDEX(L,1)                                              MAT40094
      JCOLUM=INDEX(L,2)                                            MAT40095
      DO 705 K=1,N                                                 MAT40096
      SWAP=A(K,JROW)                                               MAT40097
      A(K,JROW)=A(K,JCOLUM)                                        MAT40098
      A(K,JCOLUM)=SWAP                                             MAT40099
  705 CONTINUE                                                     MAT40100
  710 CONTINUE                                                     MAT40101
      DO 730 K = 1,N                                               MAT40102
      IF(INDEX(K,3) -1) 715,720,715                                MAT40103
 720   CONTINUE                                                    MAT40104
 730   CONTINUE                                                    MAT40105
       ID = 1                                                      MAT40106
 810  RETURN                                                       MAT40107
 715  ID = 2                                                       MAT40108
       GO TO 810                                                   MAT40109
```

ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENT

DISCRIPTION OF PARAMETERS

ST - STATION
STATION SPACING = .275 FT
B/2 - HALF THE BEAM AT THE CORRESPONDENT STATION (FT)
AMC - NONDIMENSIONAL LOCAL ADDED MASS OR ADDED MOMENT OF INERTIA
AMC = A(3,3)/(RO*(PI/2.)*(B/2.)**2) FOR HEAVE
AMC = A(4,4)/(RO*(PI/4.)*(B/2.)**4) FOR ROLL
DFC - NONDIMENSIONAL LOCAL DAMPIMG COEFFICIENT
DFC = B(3,3)/(RO*(PI/2.)*(B/2.)**2*OMEGA) FOR HEAVE
DFC = B(4,4)/((RO*(PI/4.)*(B/2.)**4)*OMEGA) FOR ROLL

1 - HEAVE (OMEGA = 8.138 SEC(-1))

ST	B/2	AMC	DFC	A(3,3)	B(3,3)
1/2	.0269	2.0346	.5091	.0058	.0141
1	.0487	1.0115	.8234	.0073	.0478
2	.1024	.5084	.9035	.0181	.2473
3	.1658	.5069	.9257	.0470	.5230
4	.2317	.5944	.8350	.0972	1.9850
5	.2934	.6324	.7373	.1658	1.5539
6	.3420	.6743	.6465	.2402	1.8515
7	.3744	.7342	.5580	.3131	1.9129
8	.3915	.8105	.4749	.3784	1.7818
9	.3956	.8824	.4145	.4210	1.5890
10	.3955	.9129	.3934	.4356	1.5088
11	.3958	.9129	.3934	.4350	1.5188
12	.3956	.9602	.3984	.4342	1.5280
13	.3958	.8458	.4420	.4036	1.6951
14	.3924	.7322	.5485	.3434	2.0509
15	.3794	.6447	.6820	.2826	2.4034
16	.3482	.6162	.8100	.2276	2.4044

17	.2891	.6221	.9432	.1584	1.9301
18	.2023	.6529	1.1211	.0814	1.1213
19	.0998	.6201	1.4349	.0244	.3499
19 1/2	.0261	3.6232	4.8805	.0075	.0815

SUM(A(3,3)) = 4.5105 LB*SEC**2/FT**2
SUM(B(3,3)) = 27.1175 LB*SEC/FT**2

A(3,3) = 1.2404 LB*SEC**2/FT
B(3,3) = 7.4573 LB*SEC/FT

2 - ROLL (OMEGA = 4.145 SEC(-1))

ST	B/2	AMC	DFC	A(4,4)	B(4,4)
1/2	.0269	6445.1500	305.6656	.0051	.0010
1	.0487	573.9870	29.1875	.0049	.0020
2	.1024	22.3790	1.2832	.0037	.0009
3	.1658	1.6605	.1154	.0021	.0005
4	.2317	.1297	.0081	.0006	.0001
5	.2934	.0074	.0001	.0001	.0000
6	.3420	.0376	.0026	.0018	.0002
7	.3742	.0674	.0028	.0028	.0003
8	.3915	.1040	.0016	.0037	.0001
9	.3958	.1487	.0000	.0056	.0000
10	.3958	.1747	.0002	.0065	.0000
11	.3958	.1747	.0002	.0065	.0000
12	.3958	.1682	.0001	.0063	.0000
13	.3958	.1240	.0014	.0046	.0001
14	.3924	.0763	.0050	.0027	.0007
15	.3794	.0857	.0121	.0027	.0016
16	.3482	.1288	.0133	.0029	.0012
17	.2891	.1754	.0052	.0019	.0002
18	.2023	.7017	.0051	.0018	.0001
19	.0998	22.3258	.6641	.0034	.0004
19 1/2	.0261	127.5578	1.3351	.0001	.0000

SUM(A(4,4)) = .0639 LB*SEC**2
SUM(B(4,4)) = .0080 LB*SEC

A(4,4) = .0176 LB*SEC**2*FT
B(4,4) = .0022 LB*SEC*FT

5 - ROLL (OMEGA = 8.290 SEC(-1))

ST	B/2	AMC	DFC	A(4,4)	B(4,4)
1/2	.0269	5819.8480	5134.764	.0046	.0339
1	.0467	4990.3570	441.6100	.0043	.0313
2	.1024	18.5370	14.6124	.0051	.0203
3	.1658	1.4157	.9331	.0016	.0089
4	.2317	.1010	.0433	.0004	.0016
5	.2934	.0074	.0011	.0001	.0001
6	.3420	.0291	.0056	.0006	.0017
7	.3742	.0592	.0053	.0018	.0015
8	.3915	.1024	.0002	.0037	.0001
9	.3958	.1508	.0035	.0056	.0011
10	.3958	.1759	.0082	.0066	.0025
11	.3958	.1759	.0082	.0060	.0025
12	.3958	.1699	.0069	.0063	.0021
13	.3958	.1249	.0004	.0047	.0001
14	.3924	.0015	.0117	.0022	.0035
15	.3794	.0528	.0513	.0017	.0134
16	.3482	.1033	.0752	.0191	.0134
17	.2891	.1730	.0472	.0018	.0042
18	.2025	.6845	.0188	.0427	.0004
19	.0998	21.6162	7.3803	.0033	.0093
19 1/2	.0261	133.5900	18.3078	.0001	.0001

SUM(A(4,4)) = .1171 LB*SEC**2
SUM(B(4,4)) = .1522 LB*SEC

A(4,4) = .0322 LB*SEC**2*FT
B(4,4) = .0363 LB*SEC*FT

APPENDIX 3

NON-LINEAR MOTION PARAMETERS

1 - DETERMINATION OF HYDRODYNAMIC COEFFICIENTS USING EMPIRICAL METHODS

1 - ADDED MASS AND ADDED MOMENT OF INERTIA

Using Ref. 20 p. 61 to 63 - Heave

$$\Delta m = \xi_1 * \rho * \pi/2 \int_0^L \xi_2 \cdot y^2 (dx) \qquad (A3.1)$$

where ρ - water density 1.94 $LB*SEC^2/FT^4$

L - lenght of the model 5.5 FT

ξ_2 - coefficient from fig. 17 $\approx$ 1.

y - contour of the waterline, aproximated by a parabola of the form

$$y = B/2 \left[1 - \frac{x}{L/2} \right]^n \qquad (A3.2)$$

B - beam of the model .79 FT

n - $C_W/(1-C_W)$

C_W - waterplane coefficient - $A_{Wl}/B*L$ = .724

ξ_1 - coefficient, calculated according to

$$\xi_1 = 1/\left[1 + .8*B/L\right] = .895 \qquad (A3.3)$$

the following form may be obtained

$$\Delta m = .772*\xi_1*\rho*L*B^2*C_W^2/(1+C_W) \qquad (A3.4)$$

Plugging in the values

$\Delta m = 1.41$ $LB*SEC^2/FT$

Being the mass of the vehicle - m 1.43 $LB*SEC^2/FT$ this result may be checked by measuring the natural period

of heaving motions and using results from the hydrostatic calculations $Z_z = - *g*A_{wl} = -194.05$ LB/FT

$$\omega_n = \left[\frac{- Z_z}{m + \Delta m} \right]^{\frac{1}{2}} = 8.29 \text{ SEC}(-1)$$

The value found for the natural undamped frequency was 8.038 (SEC-1).

Similarly for roll, the added moment of inertia is expressed as a percentage of the moment of inertia. This is found, from data reffering to full scale vehicles, acording to the formula

$$I_{xx} = W/g*((B^2*C_W^2)/11.4*C_b + D^2/12) \text{ TON*SEC}^2 \qquad (A3.5)$$

$$= .02855 \text{ LBSEC}^2$$

From fig. 24 of the above refference

$\Delta I_{xx} = .286*I_{xx}$

Therefore

$$I_{xx} + \Delta I_{xx} = .03665 \text{ LB*SEC}^2$$

From the hydrostatic calculations

$$K_\phi = - W*GM = -2.159 \text{ LB*FT}$$

The resulting natural frequency is then

$$\omega_n = 7.7 \text{ SEC}(-1)$$

while the value found in the measurements was

$$\omega_n = 4.14 \text{ SEC}(-1)$$

The values of added mass and of addes moment of inertia, calculated in Appendix 2 were :

$$A(3,3) = 1.24 \text{ LB*SEC}^2/\text{FT}$$

$$A(4,4) = .0176 \text{ LB*SEC}^2\text{*FT}$$

The value of roll added moment of inertia is seen to be quite different the results of both methods are compared.

2 - DAMPING COEFFICIENT

For heave the following expression, taken from page 153 of the above refference was used :

$$Z_W \quad -1/2\rho\, 2\pi/\lambda(L.B.C_W)^2 . \chi^2(T/\lambda) . (\tau.L/\lambda) \qquad \text{(A3.6)}$$

where the parameters are as before and also :

λ - wave lenght = 3.11 FT for ω=8.04 SEC(-1)

χ^2 - coefficient taken from (20) fig. 52 = .460

T - draft of the vehicle = .28 FT

τ - coefficient taken from (20)fig. 53 = .200

For the values above : Z_W = -2.336 LB*SEC/FT

while when calculated by the methods of Appendix 2

Z_W = -7.4543 LB*SEC/FT which is three times the value presented above.

For roll, the calculation of linear and non-linear damping coefficients were carried simultaneously, for each transverse section of the vehicle, using Equations 21, 22, 23 or 24 of page 148 of the above refference, deppending on the shape of the section. The value of the damping for the whole vehicle was obtained by summing the sectional results

over the lenght of the vehicle in a " strip theory " way.

Therefore, using the following equations the damping is calculated.

$$K_{Di} \; -1/2\rho b_i^4 \Delta l_i \left[C_{oi}(r_o, B_i/T_i)\,|p| + \frac{g(B_i/T_i - 2.8)^2 + 4}{(b_i/g)^{\frac{1}{2}} \cdot 10^3} \right] p \qquad \text{(A3.7)}$$

$$K_{Di} \; -\tfrac{1}{2}\rho b_i^4 \Delta l_i \left[C_{oi}(r_o; B_i/T_i)\,|p| + \frac{72(B_i/T_i - 3.07)^2 + 11}{(b_i/g)^{3/4} \cdot \omega^{-4} \cdot 10^3} \right] p \qquad \text{(A3.8)}$$

$$K_{Di} \; -\tfrac{1}{2}\rho b_i^4 \Delta l_i \left[C_{oi}(r_o, B_i/T_i)\,|p| + \frac{36(B_i/T_i - 2.)^2}{(b_i/g)^{3/4} \cdot \omega^{-4} \cdot 10^3} \right] p \qquad \text{(A3.9)}$$

$$K_{Di} \; -\tfrac{1}{2}\rho b_i^4 \Delta l_i \left[.57\,|p| + \frac{g(B_i/T_i - 2.82)^2\, 4}{(bi/g)^{\frac{1}{2}} \cdot 10^3} \right] p \qquad \text{(A3.10)}$$

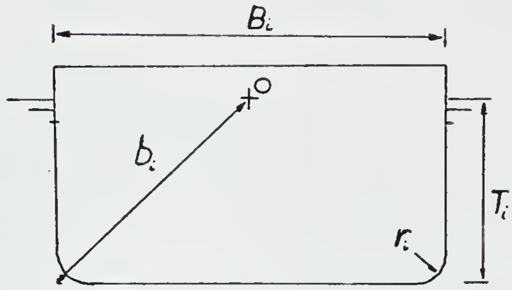

Fig. A3.1

where Eq. A3.7 and A3.8 are used for rectangular sections of different value of r_o r_i/b_i , Eq. A3.9 is used with elliptical sections and Eq A3.10 with terminal section of shape close to triangular, the following values were

found for linear and non linear damping coefficients:

$$K_p = -.00786 \text{ LB*FT*SEC} \quad - \omega = 4.145.\text{SEC}(-1)$$

$$= -.06835 \text{ LB*FT*SEC} \quad - \omega = 8.290.\text{SEC}(-1)$$

$$K_{p|p|} = -.01605 \text{ LB*FT*SEC}^2 \quad - \text{ all frequencies}$$

The values found in Apenndix 3 for the linear damping coefficients were respectively -.0022 LB*FT*SEC
-.0363 LB*FT*SEC

For the nonlinear heave damping coefficient, the values for each section were obtained by considering each section substituted by an equivalent cylinder of the same diameter when the model moves away from the water (assuming the only drag present to be separation drag) and substituted by a cylinder of the same lenght of wetted contour when the model moves into the water (assuming the only drag present to be skin friction drag) . Data from infinite cylinders was then used, the correspondent values of drag coefficient averaged over a cycle, and the results summed over the lenght of the model. The velocity entered in the calculation of the Reynolds Number was a RMS type of velocity taken from the records of the experiments.

The final value obtained was:

$$Z_{w|w|} = -1.767 \text{ LB*SEC}^2/\text{FT}^2$$

APPENDIX 4

CIRCUITRY AND DIGITIZING PROGRAM

1 - PROGRAM TO DIGITIZE DATA AND CORRESPONDENT PATCHING IN THE ANALOG COMPUTER

```
C                 PROGRAM TO DIGITIZE ROLL DATA
C                 INFORMATION RELATED TO SUBPROGRAMS USED IS
C                 AVAILABLE IN THE " HYBRID USER'S GUIDE ",
C                 PUBLISHED BY THE DEPARTMENT OF MECHANICAL ENGI-
C                 NEERING.
C     COMP  -  LOGICAL FUNCTION WHICH RETURNS A LOGICAL .TRUE. IF
C     THE ADDRESSED COMPARATOR IS IN THE  ON  STATE, IN THIS
C     PARTICULAR CASE THE COMP.  C004 .
C     COMP MUST APPEAR ON A LOGICAL STATEMENT
      LOGICAL COMP
      DATA ADDR/'C004'/
      DIMENSION K(3000),A(15)
C     COMPARATOR C004 SHOULD BE IN THE OFF STATE (CLOCK IN STOP,
C     FOR EXAMPLE ).
C     CLEAR THE ANALOG TO DIGITAL CONVERTER ( ADC )
      CALL HINT
C     THE COMPUTER IS NOW READY TO ACCEPT THE DATA
      WRITE(1,200)
  200 FORMAT( 20X,' PRESS RUN ON CONSOLE AND START THE TAPE ')
    1 IF(.NOT.COMP(ADDR))GO TO 1
      CALL RBADX(K,0,2,1000)
C     RBADX(K,IFIRST,ILAST,ICNT)  -  THIS SUBROUTINE READS A BLOCK OF  ADC
C     VALUES, EACH TIME THE  ADC  SAMPLE LINE IS PULSED. THOSE VALUES,
C     IN SCALED-FRACTION FORM ARE STORED IN THE INTEGER ARRAY  K.
C     IFIRST AND ILAST ARE, RESPECTIVELY THE FIRST AND LAST CHANNELS
C     FROM WHICH DATA IS READ. ICNT SPECIFIES THE NUMBER OF SAMPLES
C     TO BE READ
      CALL RBADX(K,0,4,1000)
C     ALL THE DATA IS NOW STORED IN  K
      WRITE(1,600)
  600 FORMAT(20X,' ADC COMPLETED ')
C     THE FOLLOWING CARD IS PRESENT TO AVOID THE LAST CARD IN THE
C     PROGRAM TO BE PUNCHED
      READ(2,100)
C     CONVERT THE DATA FROM SCALED FRACTION INTO REAL, USING THE
C     FUNCTION  FLOTS
```

```
      DO 3 I=2, 2998
      I=I-1
      DO 2 J=1, 9
      A(J)=FLOTS(K(I))
    2 I=I+1
C     PUNCH THE DATA ON CARDS
      WRITE(2,100)(A(J), J=1,9)
  100 FORMAT(9F8.4)
    3 CONTINUE
      WRITE(1,300)
  300 FORMAT( 20X, ' DATA IS DIGITIZED ')
      CALL EXIT
      END
```

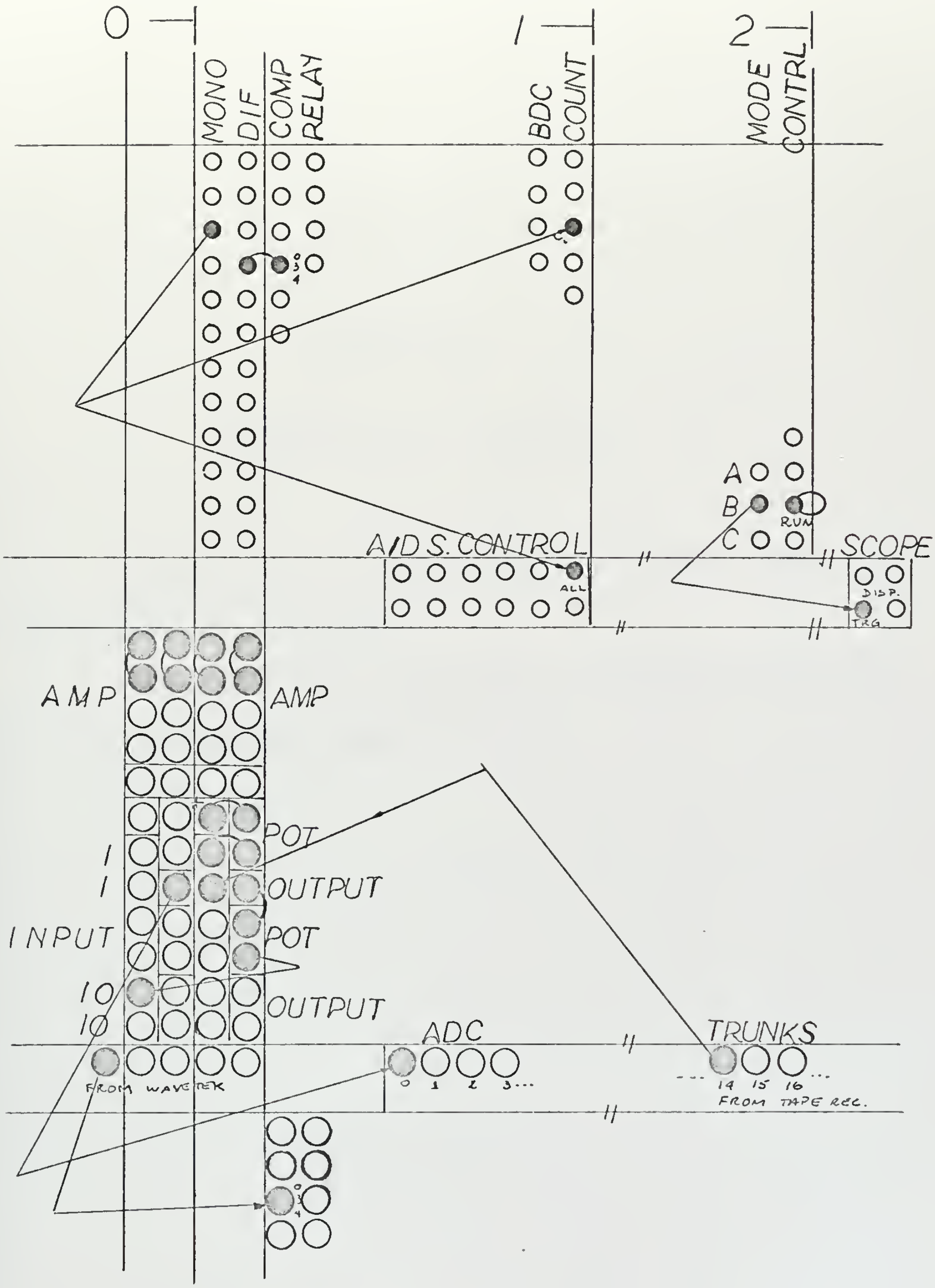

FIG. A4 - 1

EXAMPLE OF PATCHING THE 680 ANALOG COMPUTER

APPENDIX 5

IDENTIFICATION PROCEDURE

1 - DISCRIPTION OF THE COMPUTER PROGRAMS USED IN THE IDENTIFICATION PROCEDURE

2 - EXAMPLE OF THE BEHAVIOUR OF VARIABLES AND RELATED VARIANCES DURING IDENTIFICATION

```
C     MAIN FOR KALMAN FILTER  KLMN
C     DISCRIPTION OF PARAMETERS
C         XHAT  -  ESTIMATE OF STATE VECTOR X
C         Z  -  MEASUREMENT VECTOR
C         U  -  CONTROL VECTOR
C         W  -  ENVIRONMENTAL NOISE PLUS STRUCTURAL UNCERTAINTY
C         V  -  MEASUREMENT NOISE
C         NX  -  DIMENSION OF THE STATE VECTOR X AND ITS ESTIMATE XHAT
C         NZ  -  DIMENSION OF THE MEASUREMENT VECTOR Z
C         NU  -  DIMENSION OF THE CONTROL VECTOR U
C         NTIME  -  TOTAL NUMBER OF MEASUREMENTS
C         DELTA  -  TIME INTERVAL BETWEEN MEASUREMENTS
C         P  -  COVARIANCE MATRIX OF THE STATE ESTIMATE XHAT
C         DQ  -  COVARIANCE MATRIX OF W ( DIAGONAL )
C         DRINV  -  INVERSE OF THE COVARUANCE MATRIX OF V  ( DIAGONAL )
C         ZZ  -  ARRAY FOR THE STORAGE OF THE MEASUREMENTS
C         UU  -  ARRAY FOR STORAGE OF CONTROL
C         Q = SQRT(DQ/DELTA)
C         R = SQRT(DELTA/DRINV)
C         YPLOT  -  EACH OF XHAT,U AND Z FOR THE PURPOSE OF PLOTTING
C         UCAL  -  SCALING OF THE AMPLITUDE OF THE CONTROL FOR
C                  THE PURPOSE OF PLOTTING
C         SIMBL(K)  -  LABELS OF YPLOT IN THE PLOT
C         YMIN,YMAX  -  SCALING OF YPLOT FOR THE PURPOSE OF PLOTTING
C         NUMBR  -  NUMBER OF DESIRED RUNS ON THE DATA
C                   FOR EACH RUN IT IS NECESSARY TO SPECIFY
C                   A SET OF VALUES OF Q(I), R(I), P(I,I) AND XHAT(I)
C         FORMAT OF INPUT PARAMETERS
C
C     CARD 1   NUMBR                       FORMAT(I2)
C     CARD 2   NX,NU,NZ,NTIME,DELTA       FORMAT(3I2,I4,F10.5)
C     CARD 3   UCAL,YMIN,YMAX              FORMAT(3F10.5)
C     CARD4    SYMBJ(K),K=1, (NX+NU+NZ) FORMAT(20A1)
C     CARD 5   (UU(I,J),ZZ(K,J), I=1, NU , K=1,NZ),J=1  NTIME
C
C
```

```
C
C                                                 FORMAT(9F8.4)
C      CARD M     Q(I),I=1, NX                    FORMAT(10F10.5)
C      CARD M+1  R(I),I=1 NZ                      FORMAT(10F10.5)
C      CARD M+2  P(I,I),I=1 NX                    FORMAT(10F10.5)
C      CARD M+3  XHAT(I),I=1 NX                   FORMAT(10F10.5)
      DIMENSION Z(10),XHAT(10),U(10),P(10,10),DQ(10),DRINV(10),Y(10),Q(1
     .10),R(10),YPLOT(20),SYMBL(20),ZZ(10,1000),UU(10,1000)
      READ(5,1000)NUMBR
 1000 FORMAT(I2)
      READ(5,2000)NX,NU,NZ,NTIME,DELTA
 2000 FORMAT(3I2,I4,F10.5)
      NY=NX+NZ+NU
      READ(5,6002)UCAL,YMIN,YMAX
 6002 FORMAT(3F10.5)
      READ(5,6003)(SYMBL(K),K=1, NY)
 6003 FORMAT(20A1)
      DO 4 J=1, NTIME
      READ(5,3000)((UU(I,J),I=1, N1),(ZZ(I,J),I=1, NZ))
 3000 FORMAT(9F8.4)
    4 CONTINUE
      DO 1 NN=1, NUMBR
      DO 11 I=1,NX
      DO 11 J=1,NX
   11 P(I,J)=0.0
      READ(5,6001)(Q(I),I=1, NX)
      READ(5,6001)(R(I),I=1, NZ)
      READ(5,6001)(P(I,I),I=1, NX)
      READ(5,6001)(XHAT(I),I=1, NX)
 6001 FORMAT(10F10.5)
      WRITE(6,201)NX,NU,NZ,NTIME,DELTA
      WRITE(6,202)(Q(I),I=1, NX)
      WRITE(6,203)(R(I),I=1, NZ)
      WRITE(6,204)(P(I,I),I=1, NX)
      WRITE(6,205)(XHAT(I),I=1, NX)
  201 FORMAT(10X,'NX = ',I2/10X,'NU = ',I2/10X,'NZ = ',I2/7X,'NTIME = ',
```

```
     1I4/7X,'DELTA = ',F10.5)
 202 FORMAT (5H  Q  ,6F10.4)
 203 FORMAT (5H  R  ,6F10.4)
 204 FORMAT (5H  P  ,6F10.4)
 205 FORMAT (5H XHAT,6F10.4)
     CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,0)
     DO 12 I=1,NX
  12 DQ(I)=DELTA*Q(I)*Q(I)
     DO 14 I=1,NZ
  14 DRINV(I)=DELTA/(R(I)*R(I))
     DO 27 NT=1,NTIME
     DO 2 I=1, NU
   2 U(I)=UU(I,NT)
     DO 3 I=1, NZ
   3 Z(I)=ZZ(I, NT)
     CALL KLMN(Z,XHAT,U,P,DQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
     IF(DETP1)25,13,25
  25 IF(DETP2)26,13,26
  26 K=0
     DO 41 I=1,NX
     K=K+1
  41 YPLOT(K)=XHAT(I)
     DO 42 I=1,NZ
     K=K+1
  42 YPLOT(K)=Z(I)
     DO 43 I=1,NU
     K=K+1
  43 YPLOT(K)=UCAL*U(I)
     CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,1)
     IF(NT-20*(NT/20))27,47,27
  47 WRITE(7,110)NT
     WRITE(7,101)(XHAT(I), I=1, NX)
     WRITE(7,103)(Z(I), I=1, NZ)
     WRITE(7,104)(P(I,I),I=1, NX)
  27 CONTINUE
     WRITE(7,100)
```

```
      DO 31 I=1,NX
   31 WRITE(7,104)(P(I,J), J=1, NX)
  100 FORMAT (1H0)
  101 FORMAT (6H XHAT ,10F10.3)
  103 FORMAT (6H    Z ,10F10.3)
  104 FORMAT (6H    P ,10F10.3)
  110 FORMAT (4H NT=,I3)
   13 WRITE(7,4000)DETP1,DETP2
 4000 FORMAT(2F10.5)
    1 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE KLMN(Z,X,U,P,DQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
C     KALMAN FILTER
C         DISCRIPTICN OF PARAMETERS
C         THE SAME AS IN MAIN PROGRAM, EXCEPT:
C         PROVIDED BY CALLING PROGRAM
C         X  - ESTIMATE OF THE STATE BEFORE MEASUREMENT
C         P  -  COVARIANCE OF THE STATE BEFORE MEASUREMENT
C         RETURNED BY KLMN
C         X  -  ESTIMATE OF THE STATE AFTER MEASUREMENT
C         P  -  COVARIANCE OF THE STATE AFTER MEASUREMENT
C         DETP1  -  DETERMINANT OF THE MATRIX TO BE INVERTED
C         IF DETP1  =  0  -  SOLUTION IS SINGULAR
      DIMENSION Z(10),X(10),U(10),P(10,10),
     1DQ(10),DRINV(10),F(10,10),D(10,10),Y(10)
      CALL FOFXU(X,U,Y)
      DO 10 I=1,NX
   10 X(I)=X(I)+DELTA*Y(I)
      CALL FDERV(F,X,U,NX)
      DO 12 I=1,NX
      DO 12 J=1,NX
      SUM=0.0
      DO 11 K=1,NX
   11 SUM=SUM+F(I,K)*P(K,J)
   12 D(I,J)=DELTA*SUM
      DO 15 I=1,NX
      DO 14 J=I,NX
      P(I,J)=P(I,J)+D(I,J)+D(J,I)
   14 P(J,I)=P(I,J)
   15 P(I,I)=P(I,I)+DQ(I)
      DO 17 I=1,NZ
      DO 16 J=1,NZ
   16 D(I,J)=P(I,J)
   17 D(I,I)=D(I,I)+1.0/DRINV(I)
      CALL MINIO(D,NZ,DETP1)
      DO 23 I=1,NX
      DO 23 J=1,NZ
```

```
      SUM=0.0
      DO 22 K=1,NZ
   22 SUM=SUM+P(I,K)*D(K,J)
   23 F(I,J)=SUM
      DO 25 I=1,NX
      DO 25 J=1,NX
      SUM=0.0
      DO 24 K=1,NZ
   24 SUM=SUM+F(I,K)*P(K,J)
   25 C(I,J)=SUM
      DO 26 I=1,NX
      DO 26 J=1,NX
   26 P(I,J)=P(I,J)-D(I,J)
      DETP2=1.0
   18 DO 19 I=1,NZ
   19 Y(I)=Z(I)-X(I)
      DO 20 I=1,NX
      DO 20 K=1,NZ
   20 X(I)=X(I)+P(I,K)*Y(K)*DRINV(K)
   13 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PRPLT(Y,SYMBL,YMIN,YMAX,NY,NEWPL)
C          PLOTTING ROUTINE
C          PARAMETERS AS IN MAIN, EXCEPT:
C          Y  =  YPLOT
C          NY  =  NX + NZ + NU
C          NEWPL  -  CONTROL VARIABLE
C          IF NEWPL = 0  ROUTINE SCALES AND LABELS THE AXIS
C          IF NEWPL = 1  ROUTINE PLOTS
      DIMENSION A(101),Y(20),SYMBL(20)
      DATA BLANK/' '/,ASTER/'*'/
      IF(NEWPL)21,11,21
   11 NPRNT=101
      NP10=(NPRNT-1)/10+1
      NHALF=10*((NPRNT-1)/20)+1
      SCALE=NPRNT-1
      SCALE=SCALE/(YMAX-YMIN)
      N=0
      DO 12 I=1,NP10
      XI=10*(I-1)
   12 A(I)=YMIN+XI/SCALE
      PRINT 101
  101 FORMAT (1H1)
      PRINT 112,  (A(I),I=1,NP10)
  112 FORMAT (11E10.2)
      DO 14 I=1,NPRNT
   14 A(I)=BLANK
      DO 15 I=1,NP10
      J=10*(I-1)+1
   15 A(J)=ASTER
      PRINT 111,  (A(I),I=1,NPRNT)
  111 FORMAT (5X,101A1)
      RETURN
   21 N=N+1
      N5=N-5*(N/5)
      DO 22 I=1,NPRNT
   22 A(I)=BLANK
```

```
      A(1)=ASTER
      A(NHALF)=ASTER
      A(NPRNT)=ASTER
      IF(N5)25,23,25
   23 DO 24 I=1,NP10
      J=10*(I-1)+1
   24 A(J)=ASTER
   25 DO 28 K=1,NY
      IF(Y(K)-YMIN)28,26,26
   26 IF(Y(K)-YMAX)27,27,28
   27 J=SCALE*(Y(K)-YMIN)+1.5
      A(J)=SYMBL(K)
   28 CONTINUE
      IF(N5)41,42,41
   41 PRINT 111,  (A(I),I=1,NPRNT)
      RETURN
   42 PRINT 113,  N,(A(I),I=1,NPRNT)
  113 FORMAT (1X,I3,1X,101A1)
      RETURN
      END
```

```
      SUBROUTINE MIN10(A,N,D)
C     MATRIX INVERSION
      DIMENSION A(10,10),L(10),M(10)
C         SEARCH FOR LARGEST ELEMENT                                    MINVD  3
      D=1.0                                                             MINVD  4
      DO 80 K=1,N                                                       MINVD  5
      L(K)=K                                                            MINVD  6
      M(K)=K                                                            MINVD  7
      BIGA=A(K,K)                                                       MINVD  8
      DO 20 J=K,N                                                       MINVD  9
      DO 20 I=K,N                                                       MINVD 10
   10 IF(ABS(BIGA)-ABS(A(I,J)))15,20,20                                 MINVD 11
   15 BIGA=A(I,J)                                                       MINVD 12
      L(K)=I                                                            MINVD 13
      M(K)=J                                                            MINVD 14
   20 CONTINUE                                                          MINVD 15
C          INTERCHANGE ROWS                                             MINVD 16
      J=L(K)                                                            MINVD 17
      IF(J-K)35,35,25                                                   MINVD 18
   25 DO 30 I=1,N                                                       MINVD 19
      HOLD=-A(K,I)                                                      MINVD 20
      A(K,I)=A(J,I)                                                     MINVD 21
   30 A(J,I)=HOLD                                                       MINVD 22
C          INTERCHANGE COLUMNS                                          MINVD 23
   35  I=M(K)                                                           MINVD 24
      IF(I-K)45,45,38                                                   MINVD 25
   38 DO 40 J=1,N                                                       MINVD 26
      HOLD=-A(J,K)                                                      MINVD 27
      A(J,K)=A(J,I)                                                     MINVD 28
   40 A(J,I)=HOLD                                                       MINVD 29
C          DIVIDE CLOUMN BY MINUS PIVOT                                 MINVD 30
   45 IF(ABS(BIGA)-1.E-20)46,46,48                                      MINVD 31
   46 D=0.0                                                             MINVD 32
      RETURN                                                            MINVD 33
   48 DO 55 I=1,N                                                       MINVD 34
      IF(I-K)50,55,50                                                   MINVD34A
```

```
   50 A(I,K)=A(I,K)/(-BIGA)                                     MINVD 35
   55 CONTINUE                                                  MINVD 36
C        REDUCE MATRIX                                          MINVD 37
      DO 65 I=1,N                                               MINVD 38
      HOLD=A(I,K)                                               MINVD 39
      DO 65 J=1,N                                               MINVD 40
      IF(I-K)60,65,60                                           MINVD 41
   60 IF(J-K)62,65,62                                           MINVD 42
   62 A(I,J)=HOLD*A(K,J)+A(I,J)                                 MINVD 43
   65 CONTINUE                                                  MINVD 44
C        DIVIDE ROW BY PIVOT                                    MINVD 45
      DO 75 J=1,N                                               MINVD 46
      IF(J-K)70,75,70                                           MINVD 47
   70 A(K,J)=A(K,J)/BIGA                                        MINVD 48
   75 CONTINUE                                                  MINVD 49
C        PRODUCT OF PIVOTS                                      MINVD 50
      D=D*BIGA                                                  MINVD 51
C        REPLACE PIVOT BY RECIPROCAL                            MINVD 52
      A(K,K)=1.0/BIGA                                           MINVD 53
   80 CONTINUE                                                  MINVD 54
C        FINAL ROW AND COLUMN INTERCHANGE                       MINVD 55
      K=N                                                       MINVD 56
  100 K=K-1                                                     MINVD 57
      IF(K)150,150,105                                          MINVD 58
  105 I=L(K)                                                    MINVD 59
      IF(I-K)120,120,108                                        MINVD 60
  108 DO 110 J=1,N                                              MINVD 61
      HOLD=A(J,K)                                               MINVD 62
      A(J,K)=-A(J,I)                                            MINVD
  110 A(J,I)=HOLD                                               MINVD 64
  120 J=M(K)                                                    MINVD 65
      IF(J-K)100,100,125                                        MINVD 66
  125 DO 130 I=1,N                                              MINVD 67
      HOLD=A(K,I)                                               MINVD 68
      A(K,I)=-A(J,I)                                            MINVD 69
  130 A(J,I)=HOLD                                               MINVD 70
```

```
      GO TO 100                                                         MINVD 71
  150 RETURN                                                            MINVD 72
      END                                                               MINVD 73
```

```
      SUBROUTINE FOFXU(X,U,Y)
C     NONLINEAR EQUATIONS OF MOTION  -  HEAVE
C     FOR THE PURPOSE OF IDENTIFYING BOTH LINEAR AND NONLINEAR COEFFI-
C     CIENTS
C     DISCRIPTION OF PARAMETERS
C         Y(I)  -  NONDIMENSIONAL TIME DERIVATIVE OF X(I)
C              Y(1)  -  WDOT/((DRAFT/2.)*OMEGA**2)
C              Y(2)  =  ZDOT/((DRAFT/2.)*OMEGA)
C              Y(I)  =  0.0           FOR  I > 2
C         X(I)  -  NONDIMENSIONAL STATE VECTOR
C              X(I)  =  W/((DRAFT/2)*OMEGA)
C              X(2)  =  Z/(DRAFT/2)
C              X(3)  =  (Z(W)/(-Z(WDOT)+M)*(1./OMEGA)
C              X(4)  =  (Z(Z)/(-Z(WDOT)+M)*(1./OMEGA**2)
C              X(5)  =  10.*(.5*Z(W*ABS(W))/(-Z(WDOT)+M))*(DRAFT/2.)
C              X(6)  =  10.*(.5*Z(Z**2)/(-Z(WDOT)+M))*((DRAFT/2.)/OMEGA**2)
C         U(I)  -  NONDIMENSIONAL CONTROL
C              U(1)  =  ZETA/(DRAFT/2.)
C     INITIAL CONDITIONS  -  OMEGA  =  8.038 SEC(-1)
C              X(1)  =  0.0
C              X(2)  =  0.0
C              X(3)  =  -.347
C              X(4)  = -1.129
C              X(5)  =  -.930
C              X(6)  =  -.689
C
      DIMENSION Y(10),X(10),U(10)
      Y(1)=X(3)*X(1)+X(4)*(X(2)-U(1))+0.1X(5)*X(1)*ABS(X(1))+0.1*X(6)*(X
     1(2)-U(1))**2
      Y(2)=X(1)
      Y(3)=0.0
      Y(4)=0.0
      Y(5)=0.0
      Y(6)=0.0
      RETURN
      END
```

```
      SUBROUTINE FDERV(F,X,U,NX)
C     PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
C     FOR THE EQUATIONS PRESENTED ABOVE
C     DISCRIPTION OF PARAMETERS
C         F(I,J)  =  DERIVATIVE OF Y(I) WITH RESPECT TO X(J)
C         X   =  STATE VECTOR
C         U   =  CONTROL VECTOR
C         NX   =  DIMENSION OF THE STATE VECTOR
C
      DIMENSION F(10,10),X(10),U(10)
      DO 1 J=1, NX
      DO 1 J=1, NX
    1 F(I,J)=0.0
      F(1,1)=X(3)+0.2*X(5)*ABS(X(1))
      F(1.2)=X(4)+0.2*X(6)*(X(2)-U(1))
      F(1,3)=X(1)
      F(1,4)=X(2)-U(1)
      F(1,5)=0.1*X(1)*ABS(X(1))
      F(1,6)=0.1*(X(2)-U(1))**2
      F(2,1)=1.0
      RETURN
      END
```

```
      SUBROUTINE FOFXU(X,U,Y)
C     NONLINEAR EQUATIONS OF MOTION  -  ROLL
C     FOR THE PURPOSE OF IDENTIFYING BOTH LINEAR AND NCNLINEAR COEFFI-
C     DISCRIPTION OF PARAMETERS
C        Y(I)  -  NONDIMENSIONAL TIME DERIVATIVE OF X(I)
C              Y(1)  =  10.*(PDOT/OMEGA**2)
C              Y(2)  =  10.*(FIDOT/OMEGA)
C              Y(I)  =  0.0        FOR  I > 2
C        X(I)  -  NONDIMENSIONAL STATE VECTOR
C              X(1)  =  10.*(P/OMEGA)
C              X(2)  =  10.*(FI)
C              X(3)  =  100.*((K(P)/(-K(PDOT)+I(XX))/OMEGA)
C              X(4)  =  (K(FI)/(-K(PDOT)+I(XX))/OMEGA**2)
C              X(5)  =  .1*(((WEIGHT*BG)/(-K(PDOT)+I(XX)))/OMEGA**2)
C              X(6)  =  (K(P*ABS(P))/(-K(PDOT)+I(XX)))
C              X(7)  =  (.167*K(FI**3)/(-K(PDOT)+I(XX)))/OMEGA**2
C        U(I)  -  NONDIMENSIONAL CONTROL
C              U(1)  =  WAVE SLOPE
C     INITIAL CONDITIONS  -  OMEGA  =  4.145 SEC(-1)
C              X(1)  =  0.0
C              X(2)  =  0.0
C              X(3)  =  -.420
C              X(4)  =  -.989
C              X(5)  =  .274
C              X(6)  =  -.126
C              X(7)  =  .312
C
      DIMENSION Y(10),X(10),U(10)
      Y(1)=0.01*X(3)*X(1)+(X(4)-10.0*X(5))*(X(2)-U(1))+10.0*X(5)*X(2)+0.
     11*X(6)*X(1)*ABS(X(1))+0.1*(X(7)+0.1667*X(5))*(X(2)-U(1))**3-0.0167
     1*X(5)*X(2)**3
      Y(2)=X(1)
      Y(3)=0.0
      Y(4)=0.0
      Y(5)=0.0
      Y(6)=0.0
```

```
Y(7)=0.0
RETURN
END
```

```
      SUBROUTINE FDERV (F,X,U,NX)
C     PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
C     FOR THE EQUATIONS PRESENTED ABOVE
C     DISCRIPTION OF PARAMETERS
C         F(I,J)  -  DERIVATIVE OF Y(I) WITH RESPECT TO X(J)
C         X  -  STATE VECTOR
C         U  -  CONTROL VECTOR
C
      DIMENSION F(10,10),X(10),U(10)
      DO 1 I=1, NX
      DO 1 J=1, NX
    1 F(I,J)=0.0
      F(1,1)=0.01*X(3)+0.2*X(6)*ABS(X(1))
      F(1,2)=X(4)+(0.3*X(7)+0.05*X(5))*(X(2)-U(1))**2-0.05*X(5)*X(2)**2
      F(1,3)=0.01*X(1)
      F(1,4)=X(2)-U(1)
      F(1,5)=-10.0*U(1)+0.0167*(X(2)-U(1))**3-0.0167*X(2)**3
      F(1,6)=0.1*X(1)*ABS(X(1))
      F(1,7)=0.1*(X(2)-U(1))**3
      F(2,1)=1.0
      RETURN
      END
```

```
      SUBROUTINE FOFXU(X,U,Y)
C     NONLINEAR EQUATIONS OF MOTION  -  ROLL
C     FOR THE PURPOSE OF IDENTIFYING ONLY THE LINEAR TERMS IN THE
C     EQUATIONS OF MOTION
C     DISCRIPTION OF PARAMETERS
C     THE PARAMETERS ARE THE SAME AS FOR THE GENERAL CASE, EX-
C     CEPT THAT IN THESE EQUATIONS X(6) AND X(7) ARE SUBSTITU-
C     TED BY THEIR PREDICTED VALUE
      DIMENSION Y(10),X(10),U(10)
      Y(1)=0.01*X(3)*X(1)+(X(4)-10.0*X(5))*(X(2)-U(1))+10.0*X(5)*X(2)-0.
     10126*X(1)*ABS(X(1))+(0.0312+0.0167*X(5))*(X(2)-U(1))**3-0.0167*X(5
     1)*X(2)**3
      Y(2)=X(1)
      Y(3)=0.0
      Y(4)=0.0
      Y(5)=0.0
      RETURN
      END
```

```
      SUBROUTINE FDERV(F,X,U,NX)
C     PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
C     FOR THE EQUATIONS PRESENTED ABOVE
      DIMENSION F(10,10),X(10),U(10)
      DO 1 I=1, NX
      DO 1 J=1, NX
    1 F(I,J)=0.0
      F(1,1)=0.01 X(3)-0.0252*ABS(X(1))
      F(1,2)=X(4)+(0.0936+0.05*X(5))*(X(2)-U(1))**2-0.05*X(5)*X(2)**2
      F(1,3)=0.01*X(1)
      F(1,4)=X(2)-U(1)
      F(1,5)=10.0*U(1)+0.0167*(X(2)-U(1))**3-0.0167*X(2)**3
      F(2,1)=1.0
      RETURN
      END
```

```
      SUBROUTINE FOFXU(X,U,Y)
      DIMENSION Y(10),X(10),U(10)
C     LINEAR EQUATIONS OF MOTION  -  ROLL
      Y(1)=0.01*X(3)*X(1)+(X(4)-10.0*X(5))*(X(2)-U(1))+10.0*X(5)*X(2)
      Y(2)=X(1)
      Y(3)=0.0
      Y(4)=0.0
      Y(5)=0.0
      RETURN
      END
```

```
      SUBROUTINE FDERV (F,X,U,NX)
C     PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
C     FOR THE EQUATIONS PRESENTED ABOVE
      DIMENSION F(10,10),X(10),U(10)
      DO 1 I=1, NX
      DO 1 J=1, NX
    1 F(I,J)=0.0
      F(1,1)=0.01*X(3)
      F(1,2)=X(4)
      F(1,3)=0.01*X(1)
      F(1,4)=X(2)-U(1)
      F(1,5)=10.0*U(1)
      F(2,1)=1.0
      RETURN
      END
```

```
                 ROUTINES OF DIFFERENT SET UP FOR THE PURPOSE
                 SIMULATION
C      MAIN FOR KLMN
C          PARAMETERS AS ABCVE, EXCEPT:
C          XX  -  TRUE VALUE OF THE STATE VECTOR X
C          COEFW, COEFV  -  CONTROL PARAMETERS FOR THE AMOUNT
C          OF NOISE PRESENT. USUALLY  0 < COEFW, COEFV < 1
           THERE ARE NO STORAGE ARRAYS ZZ AND UU
C          FORMAT OF INPUT PARAMETERS
C
C      CARD 1   NUMBR                        FORMAT(I2)
C      CARD 2   NX,NU,NZ,NTIME,DELTA        FORMAT(3I2,I4,F10.5)
C      CARD 3   UCAL,YMIN,YMAX               FORMAT(3F10.5)
C      CARD 4   SYMBJ(K),K=1, (NX+NU+NZ) FORMAT(20A1)
C      CARD 5   Q(I),I=1, NX                 FORMAT(10F10.5)
C      CARD 6   R(I),I=1,NZ                  FORMAT(10F10.5)
C      CARD 7   P(I,I),I=1, NX               FORMAT(10F10.5)
C      CARD 8   XHAT(I),I=1,NX               FORMAT(10F10.5)
C      CARD 9   XX(I),I=1, NX                FORMAT(10F10.5)
C      CARD 10  COEFW,COEFV                  FORMAT(10F10.5)
       DIMENSION Z(10),XHAT(10),U(10),P(10,10),DQ(10),DRINV(10),
      1Y(10),XX(10),W(10),V(10),Q(10),R(10),YPLOT(20),SYMBL(20)
       READ(5,1000)NUMBR
 1000  FORMAT(I2)
       READ(5,2000)NX,NU,NZ,NTIME,DELTA
 2000  FORMAT(3I2,I4,F10.5)
       NY=NX+NZ+NU
       READ(5,6002)UCAL,YMIN,YMAX
 6002  FORMAT(3F10.5)
       READ(5,6003)(SYMBL(K),K=1, NY)
 6003  FORMAT(20A1)
       DO 1 NN=1, NUMBR
       DO 11 I=1,NX
       DO 11 J=1,NX
   11  P(I,J)=0.0
       READ(5,6001)COEFW,COEFV
```

```
      READ(5,6001)(Q(I),I=1, NX)
      READ(5,6001)(R(I),I=1, NZ)
      READ(5,6001)(P(I,I),I=1, NX)
      READ(5,6001)(XHAT(I),I=1, NX)
      READ(5,6001)(XX(I),I=1, NX)
 6001 FORMAT(10F10.5)
      WRITE(6,200)COEFW,COEFV
      WRITE(6,201)NX,NU,NZ,NTIME,DELTA
      WRITE(6,202)(Q(I),I=1, NX)
      WRITE(6,203)(R(I),I=1, NZ)
      WRITE(6,204)(P(I,I),I=1, NX)
      WRITE(6,205)(XHAT(I),I=1, NX)
      WRITE(6,206)(XX(I),I=1, NX)
  200 FORMAT(10X,' COEFW = ',F10.5/10X,' COEFV = ',F10.5)
  201 FORMAT(10X,'NX = ',I2/10X,'NU = ',I2/10X,'NZ = ',I2/7X,'NTIME = ',
     1I4/7X,'DELTA = ',F10.5)
  202 FORMAT (5H  Q  ,6F10.4)
  203 FORMAT (5H  R  ,6F10.4)
  204 FORMAT (5H  P  ,6F10.4)
  205 FORMAT (5H XHAT,6F10.4)
  206 FORMAT (5H  XX ,6F10.4)
      CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,0)
      DO 12 I=1,NX
      DQ(I)=DELTA*Q(I)*Q(I)
   12 Q(I)=COEFW*Q(I)/SQRT(DELTA)
      DO 14 I=1,NZ
      CRINV(I)=DELTA/(R(I)*R(I))
   14 R(I)=COEFV*R(I)/SQRT(DELTA)
      DO 27 NT=1,NTIME
      DO 22 I=1,NZ
      W(I)=Q(I)*RAND(-1.)
   22 V(I)=R(I)*RAND(-1.)
      CALL CNTRL(U,NT)
      CALL FOFXU(XX,U,Y)
      DO 23 I=1,NZ
   23 XX(I)=XX(I)+DELTA*(Y(I)+W(I))
```

```
      DO 24 I=1,NZ
   24 Z(I)=XX(I)+V(I)
      CALL KLMN(Z,XHAT,U,P,DQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
      IF(DETP1)25,13,25
   25 IF(DETP2)26,13,26
   26 K=0
      DO 41 I=1,NX
      K=K+1
   41 YPLOT(K)=XHAT(I)-XX(I)
      DO 42 I=1,NZ
      K=K+1
   42 YPLOT(K)=Z(I)-XX(I)
      DO 43 I=1,NU
      K=K+1
   43 YPLOT(K)=UCAL*U(I)
      CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,1)
      IF(NT-20*(NT/20))27,47,27
   47 WRITE(7,110)NT
      WRITE(7,101)(XHAT(I), I=1, NX)
      WRITE(7,102)(XX(I), I=1, NX)
      WRITE(7,103)(Z(I), I=1, NZ)
      WRITE(7,104)(P(I,I),I=1, NX)
   27 CONTINUE
      WRITE(7,100)
      DO 31 I=1,NX
   31 WRITE(7,104)(P(I,J), J=1, NX)
  100 FORMAT (1HO)
  101 FORMAT (6H XHAT ,10F10.3)
  102 FORMAT (6H   XX ,10F10.3)
  103 FORMAT (6H    Z ,10F10.3)
  104 FORMAT (6H    P ,10F10.3)
  110 FORMAT (4H NT=,I3)
   13 WRITE(7,1000)DETP1,DETP2
 1000 FORMAT(2F10.5)
      CALL EXIT
      END
```

```
      FUNCTION RAND(X)
C     RANDOM NUMBER GENERATOR
      IF(X)3,2,1
    1 IX=X
    2 RAND=0.0
      N=1
      GO TO 4
    3 RAND=-6.0
      N=12
    4 N=N-1
      IX=IX*65539
      IF(IX)5,6,6
    5 IX=IX+2147483647+1
    6 Y=IX
      RAND=RAND+Y*.4656613E-9
      IF(N)7,7,4
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CNTRL(U,NT)
C     INPUT GENERAT6R
      DIMENSION U(10)
      TT=NT
      U(1)=0.2987*SIN(6.28318*TT/20.0)
      RETURN
      END
```

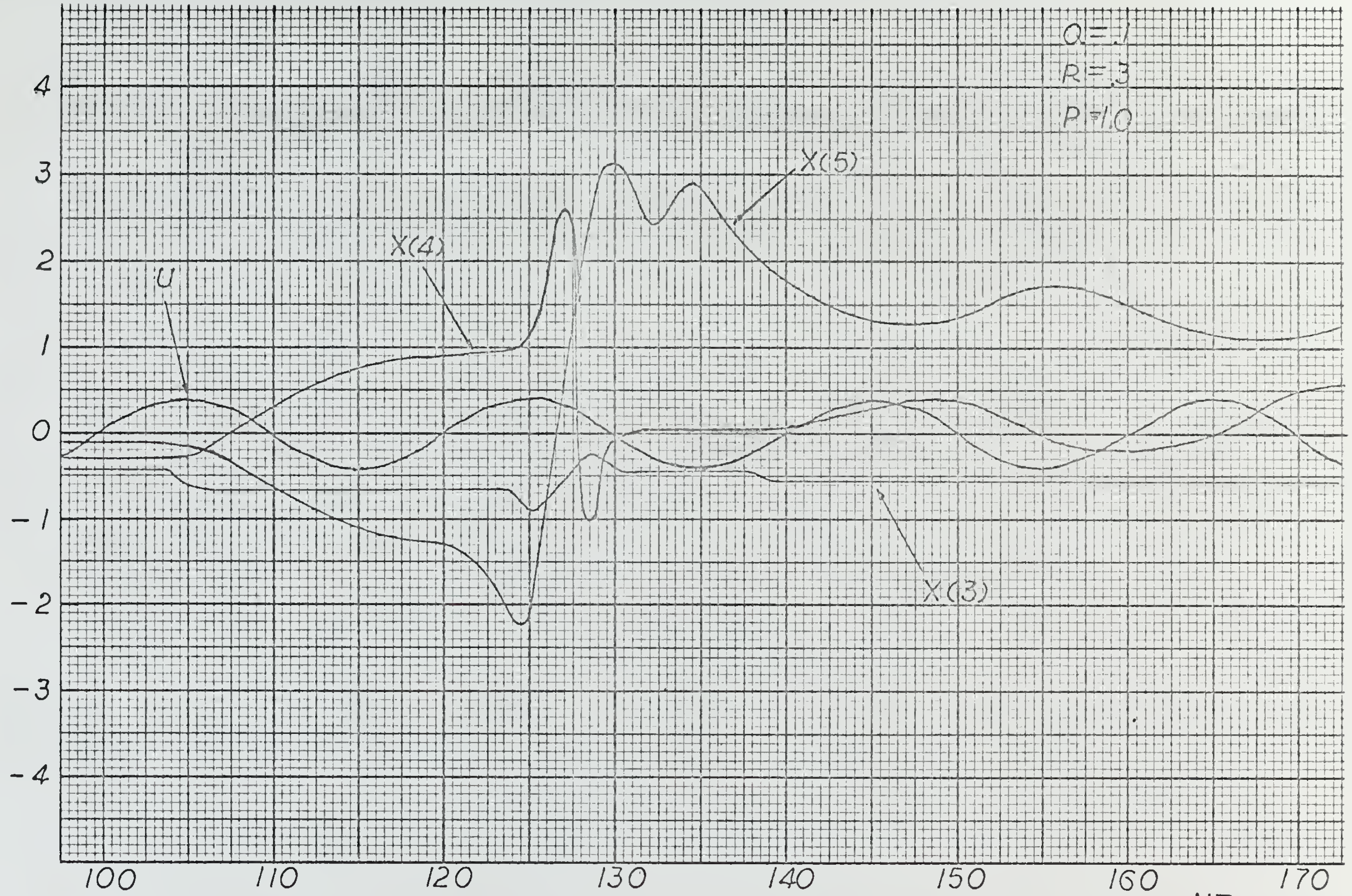

Fig. A5.1

Typical behaviour of the parameters to be identified